THE
Young Student's
MEMORIAL BOOK,
OR
POCKET LIBRARY:

CONTAINING,

I. The RUDIMENTS of Logarithms, Decimals, and Algebra, in great Variety of Rules and Caſes.

II. A very large Collection of THEOREMS and CANONS for ſolving *Queſtions* and *Problems* in the various Parts of ARITHMETIC, ALGEBRA, and FLUXIONS.

III. RULES, THEOREMS and CANONS, reſolving all the uſeful and common Problems in the MATHEMATICAL and MECHANICAL Arts and Sciences; viz. *Menſuration, Gauging, Conics, Plain and Spherical Trigonometry, Navigation, Fortification, Gunnery, Aſtronomy, Dialling, Optics, Perſpective, Architecture, Mechanics, Geometry,* &c.

IV. A large Collection of MATHEMATICAL TABLES; ſerving to various Purpoſes of *Arithmetic, Aſtronomy, Geography, Chronology,* &c. with a PERPETUAL ALMANACK, adjuſted to the preſent Year, 1735.

V. An *Apparatus* of the common MATHEMATICAL LINES, for the Operation of any *Queſtion* or *Problem*, as is perform'd by the PLAIN SCALE, &c.

By BENJAMIN MARTIN.

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THE
PREFACE.

Experience teacheth Us, that the Memory of Man is very treacherous: I have therefore endeavour'd in this little Tract to give the Young Student, in his Mathematical and Mercantile Studies; the beſt Aſſiſtance I can. For out of this he may be ſupply'd with any Rule, Canon, or Theorem, that his Memory has fail'd him in; it being a Compendium of Arts and Sciences; with an Apparatus of Mathematical Lines; whereby, with a Pair of Compaſſes, any Queſtion or Problem may be operated, as Oc-

caſion requires. My chief Study has been Uſefulneſs and Brevity; and I have corrected this ſmall Piece with ſo great Care, that the Truth of every Theorem and Figure in the Tables may be depended on: The only Errata which I think have eſcap'd, are below. If theſe my good Intentions ſhall be of Service to any, and favourably receiv'd, I have my End.*

B. M.

*ERRATA.

Pag. 8. line 21. *for* 56, read 560000. Pag. 10. line 18. read, Change all the *Signs of the Subtrahend.*

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This Plate is paſted at the End, againſt the Cover of the Book.

ARITHMETIC
BY
LOGARITHMS.

SECT. I.
Of Arithmetic *by* Logarithms.

I. LOGARITHMS are artificial Numbers in arithmetical Progreſſion, which (being fitted thereto) do expreſs the Ratio's of natural Numbers in geometrical Progreſſion.

II. Thus,

Logarithm	0.	1.	2.	3.	4.	5.	6.	*&c.*
Number	1.	2.	4.	8.	16.	32.	64.	*&c.*

Hence 'tis manifeſt, that if the Ratio of 2 to 1 be 1, then the Ratio of 4 to 1 will be 2; of 8 to 1, 3; of 64 to 1, 6; *&c.* and therefore they are fitly called (λόγων ἀριθμὸς) Logarithms, *i. e.* The Number of Ratio's or Proportions compounded, as above.

III. Hence alſo (and which is the Foundation of all the Uſe of Logarithms) we ſee that for every Multiplication and Diviſion of the Natural Numbers, there correſponds an Addition and Subſtraction of Logarithms belonging to them.

$$\text{Thus,}\left\{\begin{matrix} 1+2=3. & 2+3=5. & 2+4=6 \\ 2\times 4=8. & 4\times 8=32. & 4\times 16=64 \end{matrix}\right\}\textit{\&c.}$$

IV. This then is the great Advantage and Uſe of Logarithms; that by adding and ſubſtracting them, we ſave the labour of multiplying and dividing the Numbers of which they are the Logarithms. And by multiplying or dividing the Logarithm, by 2, 3, *&c.* you thereby Square, Cube, *&c.* or extract the Square, Cube, *&c.* Root, of its Number.

V. For this Purpoſe there are calculated Tables of Logarithms, anſwering to all the natural Numbers, from 1 to 10000, and by them may be found for any Number under 1000000, or 10000000. The Uſe of Logarithms is taught by the following Rules and Examples.

RULE I. To fit the Index to Logarithms, make it ſuch a Number as expreſſeth one leſs than the Number of Places of the Integers; and for Decimals, as below.

Thus, to theſe Numbers the Logarithms are duly fitted with Indices.

2345 = 3·370143
234·5 = 2·370143
23·45 = 1·370143
2·345 = 0·370143
.2345 = .9·370143
.02345 = .8·370143
.002345 = .7·370143
.0002345 = .6·370143

RULE II. To find the Logarithm of any Number greater than thoſe of the Table, and leſs than 10000000; ſuppoſe, for the Number 542375. Then in the Table, ſeek the Log. of the 4 firſt Figures; 5423 Log. 3·734240
Take the Logarithm of the next greater Number . . . . . . 5424 Log. 3·734320
The Difference of thoſe Logarithms. 80
Multiply by the remaining two Figures. 75
Cut off the Product ſo many Places, *viz.* two. . 60.00
And add the remainder to the firſt Log. . 3·734240
The Sum is the Log. ſought; *viz.* of 542375 = 5·734300, the Index being fitted thereto. The finding a Number to any Logarithm propoſed, is but juſt the reverſe of this Proceſs.

RULE III. To find the Logarithm of a vulgar Fraction, ſuppoſe $\frac{3}{4}$, do thus;

From the Logarithm of the Numerator. . 3=0.477121
Subſtract the Logarithm of the Denominator. 4=0.602060
The Remainder is the Logarithm of ¾=.75=.9.875061

Rule IV. To find the arithmetical Complement of any Logarithm;

Always from 10.000000
Subſtract the Logarithm, ſuppoſe 5.734300
The Remainder is the arith. Comp. *viz.* . 4.265600.

Rule V. To find the Secant of any Arch, ſuppoſe 34° 52'?

From double Radius 20.000000
Subduct the Logarithm of its Co-ſine, 55° 08' = 9.914070
There remains the Secant of . 34° 52'=10.085930

Addition of Logarithms.

I.	Multiply .	144 =	2.158362	Add.
	By . . .	12 =	1.079181	
	Product .	1728 =	3.237543	
II.	Multiply .	1385 =	3.141450	Add.
	By . . .	185 =	2.267172	
	Product	2,0225 =	5.408622	
III.	Multiply	1.385 =	0.141450	Add.
	By . .	0.0185 =	.8.267172	
	Product	.0256225 =	8.408622	
IV.	Multiply	.0144 =	.8.158362	Add.
	By . .	.0012 =	.7.079181	
	Prod.	.00001728 =	.5.237543	

Subſtraction of Logarithms.

I.	Divide . .	1728 =	3.237543	Subſ.
	By	12 =	1.079181	
	Quotient . .	144 =	2.158362	
II.	Divide .	256225 =	5.408622	Subſ.
	By	185 =	2.267172	
	Quotient .	1385 =	3.141450	

III.

III. Divide .0256225 = .8.408622 } Subſ.
By . . . . 1.385 = 0.141450 }
Quotient . .0185 = .8.267172

IV. Divide .00001728 = .5.237543 } Subſ.
By . . . . .0144 = .8.158362 }
Quotient . .0012 = .7.079181

Multiplication of Logarithms.

I. What is the Square of . . 25 = 1.397940
Multiply by 2
Anſwer . . . . 625 = 2.795880

II. What is the Cube of . . 12 = 1.079181
Multiply by 3
Anſwer . . . . 1728 = 3.237543

III. What the fourth Power of .006 = .7.778151
Multiply by 4
Anſwer . . .000000001296 = .1.112604

Diviſion of Logarithms.

I. What the Square Root of . 625 = 2.795880
Divide by 2
Anſwer . . . . 25 = 1.397940

II. What the Cube Root of 1728 = 3.237543
Divide by 3
Anſwer . . . . 12 = 1.079181

III. What the Biq. Root of } .000000001296 = .1.112604
Divide by 4
Anſwer . . . . .006 = .7.778151

Golden Rule by Logarithms.

If 13 Groſs . . . . . 13 = 1.113943 Subſ.
Coſt 7 *l.* 12 *s.* or . . 7.6 = 0.880814 } Add.
What Coſt 66½ Groſs? 66.5 = 1.822822 }
2.703636
Anſwer . . . *l.* 38.8769 = 1.589693 Rem.

Or thus, with the arithmetical Complement of the firſt Term, by Addition.

Arithmet. Compl. of . 13 = 8.886057 }
l. 7.6 = 0.880814 } Add.
Groſs 66.5 = 1.822822 }
Anſwer . . . *l.* 38.8769 = 1.589693
The ſame as before.

To find the Natural Sine, Tangent, &c.

What is the natural Sine, Tangent, or Secant of the Arch of 34° 52′?

The natural Sine, &c. is nothing but the natural Number of the Logarithm, Sine, &c.

	Logarithmetical.		Natural.
Sine	34° 52′	9.757144 =	571668, &c.
Tangent	. .	9.843074 =	696745, &c.
Secant	. .	10.085930 =	1218792, &c.

Note, the Index of the Logarithm, Sine, &c. ſuppoſes the natural Sine, &c. to be of 10 or 11 Places of Figures; but this is not neceſſary, and you may take as few as you will.

Thoſe who would know more of Logarithms, may peruſe my Syſtem of Decimal Arithmetic, where they will find ample Satisfaction.

SECT. II.

Of Decimal Arithmetic.

I. IN Decimals, the integral Quantities (whether Coin, Weight, Meaſure, Time, Motion, *&c.*) are ſuppoſed to be divided into 10 equal Parts; and each of thoſe Parts into 10 others, and ſo on, *ad infinitum.*

II. Any Integer thus divided into 10, 100, 1000, 10000, *&c.* equal Parts, becomes the Denominator to the Decimal Fraction; thus, for $\frac{2}{10}$, $\frac{7}{100}$, $\frac{35}{1000}$, *&c.* (becauſe the Denominators conſiſt of a Unit with Cyphers annexed,) we may write 0.2, 0.07, 0.035, *&c.*

III. And any Decimal Number is produced, by dividing the Numerator of a Vulgar Fraction by its Denominator; thus, $\frac{2}{5}$ is $= 0.4$; and $\frac{6}{24} = 0.25$; and $5\frac{6}{12} = 5.5$.

IV. It is to be obſerv'd, that as Cyphers annexed to whole Numbers, increaſe their Value in a Decimal or 10 fold Proportion, ſo when they are prefixed to Decimal Numbers, they decreaſe their Value in the ſame Proportion.

Integers.	Thus	10 is	Ten Units.
		100	One Hundred.
		1000	One Thouſand.
		10000	Ten Thouſand.
		100000	One Hund. Thouſand, *&c.*
Decimals.	So, vice verſâ	.1 is	One Tenth Part of an Unit.
		.01	One Hundredth Part.
		.001	One Thouſandth Part.
		.0001	A Ten Thouſandth Part.
		.00001	One Hund. Thouſand Part.
		.000001	One Millionth Part, *&c.*

V. Of Decimal Numbers, ſome are terminate, or proceed but to a certain Number of Places, and there end; as $\frac{1}{2} = .5$, $\frac{1}{4} = .25$, and $\frac{2}{16} = .125$; alſo $\frac{1}{64} = .015625$. But others are interminate or infinite, being ſuch as in their Production repeat either one or more Places of Figures; and thus they are term'd, ſingle or compound Repetends.

petends. So $\frac{1}{13}$ = .33333, *&c.* ad infinitum. So $\frac{1}{198}$ = .0050505, *&c.* and $\frac{1}{54}$ = .0185185185, *&c.* to Infinity. But 'tis uſual to daſh the firſt and laſt Figures of Repetends, thus, $.\dot{3}$; $.0\dot{0}\dot{5}$; $.0\dot{1}8\dot{5}$, or it may be $.01\dot{8}5\dot{1}$, or thus, $.018\dot{5}1\dot{8}$; there being no Occaſion to write more.

VI. The Operation of the ſour grand Rules of Addition, Subſtraction, Multiplication and Diviſion is the ſame as in whole Numbers, obſerving the following Rules, *viz.*

Rule I. In Addition and Subſtraction, place the Comma's (or Points, diſtinguiſhing the Decimal from the Integral Numbers) juſt under one another.

Rule II. In Multiplication, cut off (to the right Hand) from the Product, ſo many Figures for Decimals, as there are Decimal Places, both in the Multiplier and Multiplicand. And if there be not ſo many Places in the Product, make them ſo many, by prefixing Cyphers to the left Hand.

Rule III. In Diviſion, cut off ſo many Places of the Quotient to the Right, as there are Decimal Places more in the Dividend than in the Diviſor; but if the Quotient hath not ſo many Places, prefix Cyphers to the Left, till they are ſo many.

Rule IV. If the Dividend be leſs than the Diviſor, add Cyphers to the right Hand thereof, and thus any leſſer Number may be divided by a greater.

Addition of Decimals.

.034375	.183333	23.2375	372.5125
.139583	.229167	.734375	145.266667
.340625	2.125	42.58125	769.1375
.441667	3.234375	.936458	938.083333
.95625	5.771875	67.489583	2225.

Subſtraction of Decimals.

From	74.284	437.5	75.0034	562.
Take	45.375	89.657	57.875	93.5784
Difference	28.909	347.843	17.1284	468.4216
From	345.7578	0.547893	1.	50.
Take	157.	0.439758	0.997543	0.000375
Difference	188.7578	0.108135	0.002457	49.999625

Multiplication of Decimals.

Mult.	3.024	32.12	78.546	5745	.2365
By	2.23	24.3	436	.0675	.2435
	9072	9636	471276	28725	11825
	6048	12848	235638	40215	7095
	6048	6424	314184	34470	9460
					4730
Pro.	6.74352	780.516	34246.056	387.7875	.05758775

Mult.	57.056	.0.56879	0.03264	4572000
By	0.578	0.05674	0.02364	0.000001
Pro.	32978368	.0322731446	.0007672544	4.572

Diviſion of Decimals.

Diviſor	Dividend	Quotient
.845	295.75	35
.0078	4368	56
24.3	780.516	32.12
436	34246.056	78.546
0.534	0.30438	.57
7.684	192.100	25
7875	441.0000	560
957	7,25406	.00758
.575	,0007475	.0013
10	5784	578.4
100	5784	57.84
1000	5784	5.784
10000	5784	.5784

Note, If any deſire a compleat Knowledge of the moſt uſeful Art of Decimal Arithmetic, let them pleaſe to conſult my New Syſtem of that Art, printed for Mr. *John Noon*; and they will find plenary Satiſfaction in every part.

SECT. III.

Of Algebra.

I. ALgebra is an Art or Method of managing Arithmetical and Geometrical Computations, by Letters or Species, inſtead of Figures and Lines, or any Quantities in general.

II. In Algebra, tis pretty uſual to repreſent known or given Quantities by the firſt Letters of the Alphabet, as *a*, *b*, *c*, *d*, &c. and thoſe which are unknown or ſought by the latter Letters, as *x*, *y*, *z*, but this is entirely arbitrary.

III.

The Signs or Symbols uſed for Abbreviation ſake	Signifies
+	More; as $a+b$ is *a* more *b*, or added.
—	Leſs; as $a-b$ is *a* leſs *b*, or ſubſtracted.
×	Multiplied into; as $aa \times bc$, is *aa* multiplied into *bc*, or *aabc*.
÷	Divided by; as $aa \div bc$, is *aa* divided by *bc*, or $\frac{aa}{bc}$.
=	Equal to; as $ad = ef$, is *ad* equal to *ef*.
:	Is to; as, $a : b :: c : d$; that is, *a* is to *b*, ſo is *c* to *d*; in proportion.
::	So is;
@	Involved, Multiplied into it ſelf certain times.
ɯ	Evolved, or the Extraction of the Root.
√	The Root; as $\sqrt[2]{ab}$, is the Square Root of *ab*.

IV. Hence 'twill be very eaſy to read or expreſs an Algebraic Theorem; thus, ſuppoſe $a = 3$, $b = 5$, $d = 12$, and $z =$ the Quantity ſought; and we have the Theorem

$\frac{aa+ab-d}{b-a}=x$, this is in Numbers $\frac{3\times3+3\times5-12}{5-3}$ $=x$; or thus, $\frac{9+15-12}{5-3}=x$, that is $\frac{12}{2}=x$; or $6=x$; whence x is known.

V. The Rules to be obſerv'd in adding, ſubſtracting, multiplying and dividing Algebraic Quantities, are as follows;

I. In **Addition** there be three Caſes. Rule I. When like Quantities have like Signs, add the Co efficients (or Numbers prefix'd) together, and ſubjoin thereto the Quantities with the ſame Sign. Rule II. If the Quantities are alike, and have unlike Signs, ſubſtract the Co-efficients, and to their Difference join the Quantities with the Sign of the Greater. Rule III. When the Quantities are unlike, ſet them all down, without altering their Signs; and thence will ariſe compound Quantities.

II. In **Subſtraction** you perform all by one general Rule. Change all the Subſtrahend, and then add all the Quantities together, as by the Rules of Addition, and their Sum will be the true Remainder or Difference.

III. In **Multiplication** there are three Caſes. Rule I. When the Quantities have like Signs, and no Co-efficients, join them together with the Sign +, and that will be their Product. Rule II. If there be Co-efficients, multiply them, and to the Product join the Quantities, as before. Rule III. When the Quantities have unlike Signs, join them and the Product of the Co-efficients together, (as before) and prefix to them the Sign —.

IV. In **Diviſion** there are four Rules; Rule I. When the ſame Letters are found in both Factors, expunge them, what remains is the Quotient. Rule II. When the Quantities of the Dividend have unlike Signs to thoſe in the Diviſor, ſet down the Quotient with the Sign — prefixed. Rule III. If both Factors have Co-efficients, Divide them; and to their Quotient adjoin the Quotient Quantities. Rule IV. When the Quantities of the Diviſor are not exactly contain'd in the Dividend, then ſet them both down in the Manner of Vulgar Fractions, for the Quotient.

VI. Theſe things, I preſume, will be eaſy by the Examples which follow in all the Rules.

Addition of whole Quantities.

CASE I. Of like Quantities and like Signs.

$$\begin{array}{ll} \text{To} \;.\;. & a-2b+6d-25+5c \\ \text{Add} \;. & a-5b+2d-7 \\ \hline \text{The Sum} & 2a-7b+8d-32+5c \\ \hline \end{array}$$

CASE II. Of like Quantities having unlike Signs.

$$\begin{array}{ll} \text{To} \;.\;. & 3a-3ab+9bc+12abd-74 \\ \text{Add} \;. & -2a+ab-9bc-11abd+64 \\ \hline \text{The Sum} & a-2ab+abd-10 \\ \hline \end{array}$$

CASE III. Of unlike Quantities.

$$\begin{array}{ll} \text{To} \;. & a+b-c+4b \\ \text{Add} & -d-4f+25 \\ \hline \text{Sum} & a-d+b-4f-c+4b+25 \\ \hline \end{array}$$

An Example of all the three Caſes together.

$$\begin{array}{ll} \text{Add} \left\{ \begin{array}{l} 3aa+4abc-bb+30 \\ 2bb-3aa-2abc-25 \\ dd+2aa-3abc-3 \end{array} \right. \\ \hline \text{Sum} \quad +dd+2aa+bb-abc+2 \\ \hline \end{array}$$

Subſtraction of whole Quantities.

$$\begin{array}{lll} \text{CASE I.} & \text{From} & 2a-7b+8d-32+5c \\ & \text{Take} & a-2b+6d-25+5c \\ \hline \text{There Remains} & & a-5b+2d-7 \\ \hline \end{array}$$

Case II. From $a-2ab+abd-10$
Take $2a+ab-11abd+64-9bc$
There remains $3a-3ab+12abd-9bc-74$

Case III. From $a-d+b-4f-c+4b+25$
Take $a+b-c+4b$
There remains $-d-4f+25$

N.B. Theſe Examples of Subſtraction are only the reverſe to thoſe of Addition, and conſequently ſerve to prove them; being thus a mutual Illuſtration to each other.

Multiplication of whole Quantities.

Case I. Of Quantities having like Signs, and no Co-efficients.

Multiply	$d+b$	$-a-b$	$d+b$	$a+b$
By . .	d	$-d$	aa	ab
Products	$ad+db$	$+ad+db$	$aad+aab$	$aab+abb$

Case II. Of Quantities having like Signs and Co-efficients.

Multiply	$5a$	$-6d-a$	$8b+3d+5$
By . .	$3b$	$-5e$	6
Products	$15ab$	$+30de+5ae$	$48b+18d+30$

Case III. Of Quantities having Co-efficients, and unlike Signs.

Multiply	$+a$	$-5a+3b$	$ad-5b-10$
By . .	$-b$	$+4d$	-9
Products	$-ab$	$-20ad+12db$	$-9ad+45b+9$

Her

Here follows an Example of all thoſe Caſes promiſcuouſly.

Multiply . . . $4ab-3x+2z-4$
By . . . . . . $ab+2x-5$

$$\begin{array}{r} \hline 4aabb-3xab+2zab-4ab \\ +8xab-6xx+4zx-8x \\ -20ab+15x-10z+20 \\ \hline \end{array}$$

Pr $4aabb+5xab+2zab-24ab+7x-6xx+4zx-10z+20$

Diviſion of Whole Quantities.

Case I. $d)\,ad+db\,(a+b.$ $\quad -d)\,-ad-bd\,(a+b.$

$$\begin{array}{r} ad \\ \hline \cdot\cdot \\ db \\ db \\ \hline \cdot\cdot \end{array} \qquad \begin{array}{r} -ad \\ \hline \cdot\cdot \\ -bd \\ -bd \\ \hline \cdot\cdot \end{array}$$

Case II. When Dividend and Diviſor have unlike Signs.

$a)\,-adb\,(-db.$ $\quad -ab)\,abc+afb\,(-c-f.$

$$\begin{array}{r} -adb \\ \hline \cdots \end{array} \qquad \begin{array}{r} abc \\ \hline \cdots \\ +afb \\ +afb \\ \hline \cdots \end{array}$$

Case III. When Dividend and Diviſor have Coefficients.

$3b)\,15ab\,(5a.$ $\quad 3f)\,12af-21bf\,(4a-7b.$ $\quad -9bc)\,9bc\,(-1$

$$\begin{array}{r} 15ab \\ \hline \vdots \end{array} \qquad \begin{array}{r} 12af \\ \hline \cdot\cdot \\ -21bf \\ -21bf \\ \hline \cdots \end{array} \qquad \begin{array}{r} 9bc \\ \hline \cdots \end{array}$$

CASE IV. When the Quantities in the Diviſor are not exactly form'd in the Dividend.

$a)\,b\,(\frac{b}{a}.\quad 5d-7b)\,5b-aa\,(\frac{5b-aa}{5d-7b}.\quad 4abc)\,8adc\,(\frac{2d}{b}$

An Example of the ſeveral Caſes promiſcuouſly.

$[4zx-10z+20(ab+2x-5$

$4ab-3x+2z-4)4a^2b^2+5xab+2zab-24ab+7x-6x^2+$

$4a^2b^2-3xab+2zab-\ 4ab$

$\cdots\ 8xab\ \cdots-20ab+7x-6xx$

$[+4xz$

$8xab-6xx+4xz-8x$

$\cdots\ 20ab+15x-10z+20$

$20ab+15x-10z+20$

$\cdot\cdot\quad\cdot\cdot\quad\cdot\cdot\quad\cdot\cdot$

Involution of whole Quantities.

$2a$	$-$	$3b$	$5ab$	The Root
$4aa$	$+$	$9bb$	$25aabb$	Square
$8aaa$	$-$	$27bbb$	$125a^3b^3$	Cube
$16aaaa$	$+$	$81bbbb$	$625a^4b^4$	Biquadrate
$32aaaaa$	$-$	$243bbbbb$	$3125a^5b^5$, &c.	Surſolid, &

$a+b$ . . . . Binomial Roo

$a+b$

$a+2ab+bb$ . . . . Square

$a+b$

$aaa+3aab+3abb+bbb$ . . . . Cube

$a+b$

$a^4+4a^3b+6aabb+4ab^3+b^4$ . . Biquadrate

$a+b$

$a^5+5a^4b+10a^3bb+10aab^3+5ab^4+b^5$ Surſolid.

a —

$a - b$ . . . . Reſidual Root

$a - b$

$aa - 2ab + bb$ . . . . Square

$a - b$

$aaa - 3aab + 3abb - bbb$ . . . . Cube

$a - b$

$a^4 - 4a^3b + 6aabb - 4ab^3 + b^4$ . . Biquadrate

$a - b$

$a^5 - 5a^4b + 10a^3bb - 10aab^3 + 5ab^4 - b^5$ Surſolid.

A Theorem for raiſing the Unciæ of any Power.

Let M be the Exponent of the Power; Then $\left\{ 1 \times \frac{M-0}{1} \times \frac{M-1}{2} \times \frac{M-2}{3} \times \frac{M-3}{4} \times \frac{M-4}{5}, \text{ \&c.} \right.$

will be the Series of the Unciæ of every Term.

S E C T. IV.

Of Quantities in Arithmetical Proprotion continued.

A Series of ſuch Quantities; $0 : a : 2a : 3a : 4a : 5a : 6a :$ &c.

Or ſuppoſe $\left\{ \begin{array}{l} a = \text{Firſt Term,} \\ e = \text{The Common Exceſs,} \end{array} \right\}$ Then $a : a + e : a + 2e : a + 3e : a + 4e$, &c.

In order to ſolve Queſtions depending on theſe progreſſional Quantities; ſuppoſe it be made . . $\left\{ \right.$

A = To the firſt Term of the Series.
B = The laſt Term.
E = The Common Exceſs.
N = The Number of all the Terms.
S = The Sum of all the Series, or Terms.

Each

Each of theſe five Quantities or Parts are found by the following Theorems.

Theorem I. $\left\{ \frac{2S-NB}{N} = A. \right.$ Theor. III. $\left\{ \frac{B-A}{N-1} = E. \right.$

Theor. II. $\left\{ \frac{2S-NA}{N} = B. \right.$ Theor. IV. $\left\{ \frac{2S}{A+B} = N. \right.$

Theor. V. $\left\{ \frac{NA-NB}{2} = S. \right.$

SECT. V.

Of Quantities in **Geometrical** Proportion.

A Series of ſuch Quantities; $1 : a : aa \,.\, aaa \,.\, aaaa \,.\, aaaaa$. &c.

Or put $\left\{ \begin{array}{l} a = \text{The firſt Term} \\ e = \text{Common Ratio} \end{array} \right\}$ Then $a \,.\, ae \,.\, aee \,.\, aeee \,.\, aeeee$. &c.

For $a : ae :: ae : \frac{aaee}{a} = aee$, &c.

Let it be made again $\left\{ \begin{array}{l} A = \text{The firſt Term of the Series.} \\ B = \text{The laſt Term.} \\ E = \text{The Common Ratio.} \\ S = \text{The Sum of all the Terms.} \end{array} \right\}$ Then each Part is found by the following Theorems.

Theorem I. $S+EB-ES=A.$

Theor. II. $\left\{ \frac{ES+A-S}{E} = B. \right.$ Theor. III. $\left\{ \frac{S-A}{S-B} = E. \right.$

Theor. IV. $\left\{ \frac{BE-A}{E-1} = S. \right.$

SECT. VI.

Of Quantities in Muſical Proportion.

LET the three Quantities a, b, c, be in muſical Proportion.

Then $a : c :: a - b : b - c$.

Whence theſe Theorems, Theor. I. $\left\{ \frac{cb}{2c - b} = a. \right.$ II. $\left\{ \frac{2ac}{c + a} = b. \right.$

III. $\left\{ \frac{ba}{2a - b} = c. \right.$

Again, ſuppoſe theſe four Quantities a, b, c, d, be in muſical Proportion.

Then $a : d :: a - b : c - d$.

Whence theſe Theorems, Theor. I. $\left\{ \frac{db}{2d - c} = a. \right.$

Theor. II. $\left\{ \frac{2da - ca}{d} = b. \right.$ III. $\left\{ \frac{2da - db}{a} = c. \right.$

Theor. IV. $\left\{ \frac{ca}{2a - b} = d. \right.$

SECT. VII.

Of Proportion Disjunct; or Golden Rule Direct.

SUppoſe $a : b :: c : d$ = the fourth Term unknown.

Then $\frac{bc}{a} = d$; or $\frac{b}{a} \times c = d$; or $\frac{c}{a} \times b = d$.

But if the Proportion be Inverſe,

Then $\frac{ab}{c} = d$; or $\frac{a}{c} \times b = d$; or $\frac{b}{c} \times a = d$.

SECT. VIII.

The Double Rule of Three Direct and Inverſe.

In the Suppoſition of a Queſtion, let there be put . .
- P = Principal Cauſe of Gain, Loſs, Action, &c.
- T = To the Time, Diſtance of Place, &c.
- G = To the Gain, Loſs, or Action, &c.

And let the three Terms, wherein the Queſtion lies, be . .
- p = Principal,
- t = Time,
- g = Gain,

&c. ſought for.

When any five of theſe are given, the ſixth is found by the following

Three Theorems, *viz.*

I. $\frac{Gpt}{TP}=g$. II. $\frac{TPg}{Gt}=p$.

III. $\frac{TPg}{Gp}=t$.

N. B. Theſe Theorems find the Anſwer abſolutely, without regarding whether the Proportion be direct or reverſe.

SECT. IX.

Single Fellowſhip, or without Time.

Suppoſe
- A, B, C, D, &c. Merchants in Company.
- *a*, *b*, *c*, *d*, &c. Each Man's Part in the Stock

Then $a+b+c+d$, &c. = S, The Joint Stock.

And let G = the Gain acquired by Trading; or Loſs, &c.

The

Then, {
Theor. I. $\frac{Ga}{S} =$ A's Gain.
Theor. II. $\frac{Gb}{S} =$ B's Gain.
Theor. III. $\frac{Gc}{S} =$ C's Gain.
Theor. IV. $\frac{Gd}{S} =$ D's Gain.
} *&c.*

SECT. X.

Double Fellowſhip, or that with Time.

Suppoſe {
A, B, C, D, *&c.* Merchants trading in Company.
a, *b*, *c*, *d*, &c. Each Man's Money in Stock.
w, *x*, *y*, *z*, &c. the Time it is employ'd.

Alſo let {
$aw + bx + cy + dz$, &c. $= S$, the Sum of the Products of the Times, and Stocks.
$G =$ the whole Gain, Loſs, *&c.* by Trading.

Then it will be $S : G :: 1 : \frac{G}{S}$ whence the following

Theorems, *viz.* {
Theor. I. $\frac{G}{S} \times aw =$ A's Gain.
Theor. II. $\frac{G}{S} \times bx =$ B's Gain.
Theor. III. $\frac{G}{S} \times cy =$ C's Gain.
Theor. IV. $\frac{G}{S} \times dz =$ D's Gain.
} *&c.*

SECT.

SECT. XI.

Alligation Medial.

Suppoſe { A, B, C, D, *&c.* The Quantities to be mixed.
a, *b*, *c*, *d*, &c. The particular Rates or Values.

Then { $A+B+C+D=S$, The Sum of all the Quantities.
$a+b+c+d=R$, The Sum of all the Rates.

Then it will be, As $S : R ::$ { $1 : \frac{R}{S} =$ / $2 : \frac{2R}{S} =$ / $3 : \frac{3R}{S} =$ } Mean Rate of { 1 Part. / 2 Parts. / 3 Parts, *&c.*

SECT. XII.

Alligation Alternate.

Let	The Difference between	
a =	First	Rate and the Laſt.
b =	Second	
c =	Third	
d =	Fourth	
x =	Mean	
P =	Mean	Rate and Second
q =	Firſt	
a =	Firſt	Rate and the Penult.
b =	Second	
c =	Third	
d =	Fourth	
z =	Mean	
n =	Penult, and the laſt Rate.	
S =	Sum of all the Quantities to be mixed.	

Note; By this Method of raiſing Theorems, this Part of Arithmetic, call'd Alligation Alternate, is brought to its utmoſt Perfection, as giving all poſſible Anſwers to any Queſtion; and ſuch Theorems are no where elſe to be found, that I know of.

If there be two Quantities A and B, to be mixed; you have theſe Theorems;

Theor. I. $\left\{\frac{Sx}{a}=A.\right.$ II. $\left\{\frac{Aa}{x}=S.\right.$ III. $\left\{S-A=B.\right.$

If there be three Quantities or Ingredients, A, B, C; then you have

Theor. I. $\left\{\frac{Sx}{a}=A\right.$ Greateſt. III. $\left\{\frac{Sx-Aa}{n}=B.\right.$

Theor. II. $\left\{\frac{Sp}{q}=A\right.$ Leaſt. IV. $\left\{S-A-B=C.\right.$

If there are four Quantities or Ingredients, A, B, C, D; you have

Theor. I. $\left\{\frac{Sx}{a}=A\right.$ Greateſt. III. $\left\{\frac{Sx-Aa}{b}=B\right.$ Greateſt.

Theor. II. $\left\{\frac{Sp}{q}=A\right.$ Leaſt. IV. $\left\{\frac{Sz-Aa}{b}=B\right.$ Leaſt.

Theor. V. $\left\{\frac{Sx-Aa-Bb}{n}=C.\right.$

If there be five Ingredients, A, B, C, D, E; you have the greateſt and leaſt Quantity of A and B, as before; then

Theor. I. $\left\{\frac{Sx-Aa-Bb}{c}=C\right.$ Greateſt.

Theor. II. $\left\{\frac{Sz-Aa-Bb}{c}=C\right.$ Leaſt.

Theor. III. $\left\{\frac{Sx-Aa-Bb-Cc}{n}=D.\right.$

If there be ſix Quantities, A, B, C, D, E, F; you find the greateſt and leaſt Quantities of A, B, and C, as before; then

Theor. I. $\left\{\frac{Sx-Aa-Bb-Cc}{d}=D\right.$ Greateſt.

Theor. II. $\left\{ \frac{Sz - Aa - Bb - Cc}{d} = D \right.$ Leaſt.

Theor. III. $\left\{ \frac{Sx - Aa - Bb - Cc - Dd}{x} = E. \right.$

And thus you may proceed to raiſe Theorems for any Number of Quantities you pleaſe, or have occaſion for.

The Rule of Double Poſition.

Let it be
A = The Number or Quantity ſought.
B = The Firſt } ſuppoſed Number.
C = The Second }
D = The Firſt } Errour.
E = The Second }

Then if the Errours be both alike, *viz.* both too great, or both too little; then you have this

Theorem, $\left\{ \frac{DC - EB}{D - E} = A, \right.$ **the Number ſought.**

But if the Errours be unlike; that is, one too great, the other too ſmall; you will have this

Theorem, $\left\{ \frac{EB + DC}{E + D} = A. \right.$

SECT. XIII.

Simple Intereſt.

Let
P = The Principal put to Intereſt.
R = The Ratio of the Rate, *per Cent. per Ann*
T = The Time it continues at Intereſt.
A = The Amount of the Principal and Intereſt.

Then Theor. I. $TPR + P = A$. II. $\left\{ \frac{A}{TR + 1} = P. \right.$

$$\text{Theor. III.}\left\{\frac{A-P}{RP}=T.\right. \qquad \text{IV.}\left\{\frac{A-P}{TP}=R.\right.$$

SECT. XIV.

Annuities in Arrears, Simple Intereſt.

Let
- U = Annuity's, Penſion, yearly Rent, &c.
- T = The Time of its being unpaid, or forborn.
- R = The Ratio of Intereſt, as before.
- A = The Amount of the Annuity and its Intereſt.

Alſo let $\frac{2}{R}-1=x$; then you have the following Theorems;

$$\text{Theor. I.}\left\{\frac{TTRU-TRU+2TU}{2}=A.\right.$$

$$\text{Theor. II.}\left\{\frac{2A}{TTR-TR+2T}=U.\right.$$

$$\text{Theor. III.}\left\{\frac{2A-2TU}{TTU-TU}=R.\right.$$

$$\text{IV. } T=\sqrt{\frac{2A}{RU}+\frac{xx}{4}}:-\tfrac{1}{2}x.$$

SECT. XV.

The preſent Worth of Annuities, Simple Intereſt.

Suppoſe the ſame Letters, U, T, R, P, ſignify the ſame Things as before; then we have theſe Theorems;

$$\text{Theor. I.}\left\{\frac{TTR-TR+2T}{2TR+2}\times U=P.\right.$$

$$\text{Theor. II.}\left\{\frac{2PTR+2P}{TTR-TR+2T}=U.\right.$$

Theor. III. $\left\{ \frac{2P-2TU}{TTU-TU-2PT}=R. \right.$

$$\left\{ \begin{array}{l} \text{Let } \frac{2}{R}-\frac{2P}{U}-1=x. \\ \text{Then will } TT\pm xT=\frac{2P}{RU}. \end{array} \right.$$

Hence, Theor. IV. $\left\{ \sqrt{\frac{2P}{RU}+\frac{xx}{4}}\pm\frac{1}{2}x=T. \right.$

Note, P is here the preſent Worth of the Annuity, o Purchaſe Money.

SECT. XVI.

Compound Intereſt.

Let
P = The Principal put to Intereſt.
t = The Time of its Continuance.
A = The Amount of the Principal, and its Intereſt
R = The Amount of one Pound, and its Intereſt for one Year, at any given Rate, *per Cent* *per Annum.*

Theor. I. $PR^t=A$. II. $\left\{ \frac{A}{R^t}=P. \right.$ III. $\left\{ \frac{A}{P}=R^t. \right.$

SECT. XVII.

Annuities, &c. in Arrear; Compound Intereſt.

HEre R is the ſame as in the laſt, and alſo t; b U and A ſignify the ſame Things as under thi Head of Simple Intereſt.

Theor.

$$\text{Theor. I.}\left\{\frac{UR^t-U}{R-1}=A.\right. \qquad \text{II.}\left\{\frac{RA-A}{R^t-1}=U.\right.$$

$$\left\{\text{Let}\,\frac{A}{U}=x.\right. \qquad \text{III.}\left\{\frac{RA+U-A}{U}=R^t.\right.$$

$$\text{Theor. IV.}\left\{\frac{A-U}{U}=xR-R^t.\right.$$

SECT. XVIII.

The **preſent Worth** of Annuities, Compound Intereſt.

HEre P is the preſent Worth, or Purchaſe Money, as before: Then,

$$\text{Theor. I.}\left\{\frac{U-\frac{U}{R^t}}{R-1}=P.\right. \qquad \text{II.}\left\{\frac{RPR^t-PR^t}{R^t-1}=U.\right.$$

$$\left\{\text{Let}\,\frac{U}{P}=x.\right. \qquad \text{III.}\left\{\frac{U}{P+U-PR}=R^t.\right.$$

$$\text{Theor. IV.}\left\{\frac{U}{P}=xR^t+R^t-R^t+1.\right.$$

Note, Thoſe Equations of Compound Intereſt are beſt reſolved by the Logarithms; or the Theorems of adfected Equations, hereafter.

SECT. XIX.

Of **Freehold** Eſtates, purchaſed at Compound Intereſt.

BEcauſe Freehold Eſtates are ſuppoſed to be bought and ſold for ever, therefore the Time, *viz.* t, is out of the Queſtion; the other Letters ſignify as before; and are thus found.

Theor.

Theor. I. $\left\{ PR - P = U. \right.$ II. $\left\{ \frac{U}{R-1} = P. \right.$

Theor. III. $\left\{ \frac{P+U}{P} = R. \right.$

SECT. XX.

Extraction of Roots of Simple Equations.

In the Square Root Let . . $\left\{ \begin{array}{l} G = aa \\ R = \end{array} \right.$ $\left\{ \text{a Number, near the true Root a may be.} \right.$ $e =$ $\left\{ \text{the unknown Part of the Roo sought.} \right.$

Then . . $R + e = a$, the Root.

Alſo let $\frac{G - RR}{2} = D$. Then we have

Theor. I. $\left\{ \frac{D}{R + \frac{1}{2}e} = e. \right.$

But if $R - e = a$; then we have

Theor. II. $\left\{ \frac{D}{R - \frac{1}{2}e} = e. \right.$

For the Cube Root.

Let $\left\{ \begin{array}{l} G = aaa \\ R + e = a \\ \frac{G - R^3}{3R} = D; \end{array} \right.$ Then we have

Theor. I. $\left\{ \frac{D}{R + e} = e. \right.$

But if $R - e = a$; it will be,

Theor. II. $\left\{ \frac{D}{R - e} = e. \right.$

For the Biquadratic Root.

$$\text{Let}\begin{cases} G = aaaa \\ R + e = a \\ \dfrac{G - R^4}{2RR} = D, \text{ we have} \end{cases}$$

$$\text{Theor. I.}\left\{ \frac{D}{2R + 3e} = e. \right.$$

But if $R - e = a$, we have

$$\text{Theor. II.}\left\{ \frac{D}{2R - 3e} = e. \right.$$

For the Surſolid Root.

$$\text{Let}\begin{cases} G = aaaaa \\ R + e = a \\ \dfrac{G - R^5}{5R^3} = D; \text{ we have} \end{cases}$$

$$\text{Theor. I.}\left\{ \frac{D}{R + 2e} = e. \right.$$

If $R - e = a$; we then have

$$\text{Theor. II.}\left\{ \frac{D}{R - 2e} = e. \right.$$

SECT. XXI.

The Solution of Affected Quadratick Equations.

CASE I. Suppoſe $aa + 2ba = x$; Then is $a = \sqrt{bb + x} : - b$.

CASE II. If it be $aa - 2ba = x$; Then will $a = b + \sqrt{bb + x}$.

CASE III. But if $2ba - aa = x$; We have $a = b - \sqrt{bb - x}$.

Otherwiſe thus;

$$\text{Case I. Let}\left\{\begin{array}{l} G = aa + 2ba \\ R + e = a \\ \frac{1}{2}G - \frac{1}{2}R^2 - bR = D \end{array}\right\}\text{Then you have}$$

$$\text{Theor. I.}\left\{\frac{D}{R + b + \frac{1}{2}e} = e.\right.$$

$$\text{Case II. Let}\left\{\begin{array}{l} G = aa - 2ab \\ R + e = a \\ \frac{1}{2}G - \frac{1}{2}R^2 + bR = D \end{array}\right\}\text{Then}$$

$$\text{Theor. II.}\left\{\frac{D}{R - b + \frac{1}{2}e} = e.\right.$$

$$\text{Case III. Let}\left\{\begin{array}{l} G = 2ba - aa \\ R + e = a \\ \frac{1}{2}G + \frac{1}{2}R^2 - bR = D \end{array}\right\}\text{Then}$$

$$\text{Theor. III.}\left\{\frac{D}{b - R - \frac{1}{2}e} = e.\right.$$

SECT. XXII.

The Solution of Adfected Cubick Equations.

$$\text{Case I. Let}\left\{\begin{array}{l} G = aaa + ba \\ R + e = a \\ \frac{G}{3R} - \frac{1}{3}R^2 - \frac{1}{3}b = D \end{array}\right\}\ \text{Theor. I.}\left\{\frac{D}{R + \frac{b}{3R} + e} = e\right.$$

$$\text{Case II. Let}\left\{\begin{array}{l} G = aaa - ba \\ R + e = a \\ \frac{G}{3R} + \frac{1}{3}b - \frac{1}{3}R^2 = D \end{array}\right\}\ \text{Theor. II.}\left\{\frac{D}{R - \frac{b}{3R} + e} = e\right.$$

$$\text{Case III.}\left\{\begin{array}{l} G = ba - aaa \\ R + e = a \\ \frac{G}{3R} + \frac{1}{3}R^2 - \frac{1}{3}b = D \end{array}\right\}\ \text{Theo. III.}\left\{\frac{D}{\frac{b}{3R} - R - e} = e\right.$$

Fo

For Equations farther Adfected.

Cubic. Let $\begin{cases} G = aaa + baa + ca \\ R + e = a; \text{ Then} \end{cases}$

Theor. I. $\left\{ \frac{G - R^3 - bR^2 - cR}{3R^2 + 2bR + c} = e. \right.$

Biquadratic. Let $\begin{cases} G = a^4 + ba^3 + ca^2 + da \\ R + e = a; \text{ Then} \end{cases}$

Theor. II. $\left\{ \frac{G - R^4 - bR^3 - cR^2 - dR}{4R^3 + 3bR^2 + 2cR + d} = e. \right.$

From a View of theſe Theorems, 'twill be eaſy to conceive how to raiſe others for Equations, higher and moſt adfected.

SECT. XXIII.

he Powers of the Nine Digits to the Square Cube.

oots.	2	3	4	5	6	7	8	9
quare.	4	9	16	25	36	49	64	81
ube.	8	27	64	125	216	343	512	729
iquad.	16	81	256	625	1296	2401	4096	6561
urſolid.	32	243	1024	3125	7776	16807	32768	59049
q.Cube.	64.	729.	4096.	15625.	46656.	117649	262144	531441

umerical } Exponents of Fractional Powers { $a^{\frac{1}{2}}$. a $a^{1\frac{1}{2}}$. a^2. $a^{2\frac{1}{2}}$. a^3. $a^{3\frac{1}{2}}$. a^4, &c.

eral } Exponents of Fractional Powers { $a^{\frac{m}{n}}$. $a^{\frac{2m}{n}}$. $a^{\frac{3m}{n}}$. $a^{\frac{4m}{n}}$. $a^{\frac{5m}{n}}$, &c.

SECT. XXIV.

Dr. *Halley*'s Univerſal Analytical Speculum

1. For pure Powers or Simple Equations.

IF you have $aa \pm b$, $a^3 \pm b$, $a^4 \pm b$, &c. $=$ to an Number propoſed; then $a =$ the Root of the Powe neareſt to the propoſed Number; and $b =$ the Exceſs o Defect, by which it (*i. e.* the propoſed Number) exceed or comes ſhort of the ſaid neareſt Power. Thus, ſuppoſ $aaa + b = 231$, the neareſt Cube to that is 216, whoſ Root $6 = a$; and therefore $231 - 216 = 15 = b$. No to find a the Root of any propoſed Equation or Power Dr. *Halley* has given us two Forms of Theorems, the on Rational, the other Irrational, as in the Paradigm ſubjoin'd

Roots.	Ration. Form	Irration. Form.	Power
$\sqrt[2]{aa \pm b} =$	$a \pm \frac{ab}{2aa \pm \frac{1}{2}b}$	$\sqrt{aa \pm b}$	2d Pow
$\sqrt[3]{aaa \pm b} =$	$a \pm \frac{ab}{3aaa \pm b}$	$\frac{1}{2}a + \sqrt{\frac{1}{4}aa \pm \frac{b}{3a}}$	3d Pow
$\sqrt[4]{aaaa \pm b} =$	$a \pm \frac{ab}{4aaaa \pm \frac{3}{2}b}$	$\frac{2}{3}a + \sqrt{\frac{1}{9}aa \pm \frac{b}{6aa}}$	4th Po
$\sqrt[5]{a^5 \pm b} =$	$a \pm \frac{ab}{5a^5 \pm 2b}$	$\frac{3}{4}a + \sqrt{\frac{1}{16}aa \pm \frac{b}{10a^3}}$	5th Po
$\sqrt[6]{a^6 \pm b} =$	$a \pm \frac{ab}{6a^6 \pm \frac{5}{2}b}$	$\frac{4}{5}a + \sqrt{\frac{1}{25}aa \pm \frac{b}{15a^4}}$	6th Po
$\sqrt[7]{a^7 \pm b} =$	$a \pm \frac{ab}{7a^7 \pm 3b}$	$\frac{5}{6}a + \sqrt{\frac{1}{36}a \pm \frac{b}{21a^5}}$	7th Po
$\sqrt[8]{a^8 \pm b} =$	$a \pm \frac{ab}{8a^8 \pm \frac{7}{2}b}$	$\frac{6}{7}a + \sqrt{\frac{1}{49}aa \pm \frac{b}{28a^6}}$	8th Po
$\sqrt[9]{a^9 \pm b} =$	$a \pm \frac{ab}{9a^9 \pm 4b}$	$\frac{7}{8}a + \sqrt{\frac{1}{64}aa \pm \frac{b}{36a^7}}$	9th Po
$\sqrt[10]{a^{10} \pm b} =$	$a \pm \frac{ab}{10a^{10} \pm \frac{9}{2}b}$	$\frac{8}{9}a + \sqrt{\frac{1}{81}aa \pm \frac{b}{45a^8}}$	10thPo

Thus you have two Forms for extracting the Roots of all ſimple or pure Equations to the tenth Power incluſive. And within theſe two Forms the Root is ever found, being ſomewhat nearer to the Irrational than to the Rational one; and Note, if it be $+b$, the irrational Form gives the Root a little too great, and the Rational a little too ſmall; and contrarywiſe if it be $-b$.

II. For the Roots of Adfected Equations.

Suppoſe we have the adfected Equation $a^5 + ba^4 + ca^3 + da^2 + fa = G$; and imagine the Root $a = r \pm e$; then involving this Binomial Root to fifth Power of a, the ſeveral Powers with their Co-efficients are to be ranged as in the following Table.

Let the Sum of the Parts in the firſt Column $-$ G, be equal to q; the Sum of the Co-efficients of e, be = s; of ee, be = t; of eee, be = tt, *&c.*

N. B. The following Table is continued to the ſeventh Power of a, which is as far as is neceſſary.

Tabula

Tabula Poteſtatum.

$$\begin{array}{|lcccccccc}
ba^7 & =br^7 & \pm 7br^6e & +21br^5ee & \pm 35br^4e^3 & +35br^3e^4 & \pm 21br^2e^5 & +7bre^6 & \pm be^7 \\
ca^6 & =cr^6 & \pm 6cr^5e & +15cr^4e^2 & \pm 20cr^3e^3 & +15cr^2e^4 & \pm 6cre^5 & +ce^6 & : \\
da^5 & =dr^5 & \pm 5dr^4e & +10dr^3e^2 & \pm 10dr^2e^3 & +5dre^4 & \pm de^5 & : & : \\
fa^4 & =fr^4 & \pm 4fr^3e & +6fr^2e^2 & \pm 4fre^3 & +fe^4 & : & : & : \\
ga^3 & =gr^3 & \pm 3gr^2e & +3gree & \pm ge^3 & : & : & : & : \\
ha^2 & =hr^2 & \pm 2hre & +hee & : & : & : & : & : \\
ka & =kr & \pm ke & : & : & : & : & : & : \\
 & -G & : & : & : & : & : & : & : \\
 & \| & \| & \| & \| & \| & \| & \| & \| \\
\hline
\pm & q & s & t & u & w & x & y & z \\
\hline
\end{array}$$

If in the Solution, it comes out $+q$, then r is taken too big, and it will be $r-e=a$; and vice verſâ. When q and t have the ſame Sign, we ſhall have $a=r+\frac{sq}{ss-tq}$ in the rational Form; and in the Irrational it will be $a=r-\frac{\frac{1}{2}s-\sqrt{\frac{1}{4}ss-qt}}{t}$. But when q and t have different Signs, the two Forms will be $a=r+\frac{sq}{ss+tq}$; and $a=r+\frac{\sqrt{qt+\frac{1}{4}ss}-\frac{1}{2}s}{t}$. The Root a thus formed, may be further corrected in the irrational Form, by ſubſtracting therefrom the Quantity $\frac{\frac{1}{2}ue^3+\frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss+tq}}$, if it be $+e$; but if it be $-e$, add to the ſaid Root this Quantity with the Signs changed thus . . . . . $\left.\right\}\frac{\frac{1}{2}ue^3-\frac{1}{2}e^4}{\sqrt{\frac{1}{4}ss-tq}}$.

SECT. XXV.

Sir Iſaac Newton's Univerſal Theorem.

I. For raiſing the Powers of Binomial Quantities.

SUppoſe the Binomial Quantity $a+b$, be to be raiſed to the Power m;

Then let $\left\{\begin{array}{l} a=P \\ \frac{b}{a}=Q \end{array}\right\}$ Then will the Power of the Binomial $\overline{a+b}^{m}=P^{m}+\frac{m}{1}AQ+\frac{m-1}{2}BQ+\frac{m-2}{3}CQ+\frac{m-3}{4}DQ$, &c.

Alſo put $\left\{\begin{array}{l} P^{m}=A, \\ \frac{m}{1}AQ=B, \\ \frac{m-1}{2}BQ=C, \\ \frac{m-2}{3}CQ=D, \text{ \&c.} \end{array}\right\}$ II. For Extraction of the Root of any Power of any Binomial, as $\sqrt[n]{\overline{a+b}^{m}}$. we ſhall have the ſame Theorem, only with this obvious Variation;

viz. $\sqrt[n]{\overline{a+b}^{m}}=P^{\frac{m}{n}}+\frac{m}{n}AQ+\frac{m-n}{2n}BQ+\frac{m-2n}{3n}CQ+\frac{m-3n}{4n}DQ$, &c. And theſe Theorems equally ſerve for raiſing the Power or extracting the Root of any Infinite Series: as $a+bz+czz+dzzz+ez^4$, &c. by putting $bz+czz+dz^3+ez^4$, &c. $=x$; for then 'twill be $a+x=P^m$, &c. as before.

SECT. XXVI.

Of Fluxions.

The conſtant Quantities are repreſented by	a,	b,	c,	d,	&c
The flowing ones by the laſt Letters, as	v,	x,	y,	z.	
Their firſt Fluxions are thus noted	$\dot{v}$	$\dot{x}$	$\dot{y}$	$\dot{z}$.	
Their ſecond Fluxions thus . .	$\ddot{v}$	$\ddot{x}$	$\ddot{y}$	$\ddot{z}$.	
Third Fluxions thus	$\dddot{v}$	$\dddot{x}$	$\dddot{y}$	$\dddot{z}$	

The Fluxion of a Fraction $\frac{x}{y}$ is . $\frac{x}{y}$

The Fluxion of a Surd $\sqrt{\overline{x+y}}$ is $\sqrt{\;}\frac{\overline{\quad y}.\overline{\quad}}{x+y}.$

A General RULE for finding the Fluxions of flowing Quantities.

I. Multiply each Term of the Equation by the Exponent of the Power of every flowing Quantity included therein.

II. In doing this, ſtrike off the Root of any Power of every flowing Quantity, and ſubſtitute its Fluxion in its ſtead.

III. Then connect thoſe ſeveral Products into one Sum by proper Signs, and that will be the Fluxion required.

Then let it be required to find the Fluxions of theſe flowing Quantities.

x

$x+y+z$

xy

xyz

$ax+by+cz$

$ab+bx+yz$

$xxx-yyx+aaz-b^2$

By the Rule above they will be found as here adjoin'd, *viz.*

$$\begin{cases} \dot{x} \\ \dot{x}+\dot{y}+\dot{z} \\ x\dot{y}+y\dot{x} \\ xy\dot{z}+xz\dot{y}+yz\dot{x} \\ a\dot{x}+b\dot{y}+c\dot{z} \\ b\dot{x}+y\dot{z}+z\dot{y} \\ 3xx\dot{x}-yy\dot{x}-2xy\dot{y}+aa\dot{z} \end{cases}$$

Let m = the Index or Exponent of any Power of a Quantity x. Then univerſally the Fluxion of the Power x^m, will be $mx^{m-1}\dot{x}$.

Thus, let the Fluxionary Powers be given, as here adjoin'd.

$$\begin{cases} xx \\ xxx \\ xx+yy \\ xxyy \\ x^4+y^4 \\ 2xxxxx \end{cases}$$

Their Fluxions will be

$$\begin{cases} 2x\dot{x} \\ 3xx\dot{x} \\ 2x\dot{x}+2y\dot{y} \\ 2x\dot{x}yy+2y\dot{y}xx \\ 4x^3\dot{x}+4y^3\dot{y} \\ 10xxxx\dot{x} \end{cases}$$

Let
N = Numerator.
D = Denominator.
n = Fluxion of Numer.
d = Fluxion of Denominator.

Then $\dfrac{\mathrm{D}n-\mathrm{N}d}{\mathrm{DD}}$ = Fluxion of any Fractions, *viz.*

$$\begin{cases} \dfrac{x}{y} \\ \dfrac{a}{x} \\ \dfrac{x}{a} \\ \dfrac{x}{a+x} \\ \dfrac{z+y}{x} \end{cases}$$

For by the ſaid Theorem, the Fluxions of thoſe Fractions will be

$$\left\{\begin{array}{l} \left\{\dfrac{y\dot{x}-x\dot{y}}{yy}\right. \\[2ex] \left\{\dfrac{-a\dot{x}}{xx}\right. \\[2ex] \left\{\dfrac{a\dot{x}}{xx}\right. \\[2ex] \left\{\dfrac{a\dot{x}}{aa+2ax+xx}\right. \\[2ex] \left\{\dfrac{x\dot{z}+x\dot{y}-z\dot{x}-v\dot{x}}{xx}\right. \end{array}\right.$$

Alſo, becauſe $\sqrt[2]{y}$, $\sqrt[3]{y}$, $\sqrt[4]{y}$, &c. are of the ſame Value with thoſe Fractional Powers $a^{\frac{1}{2}}$, $a^{\frac{1}{3}}$, $a^{\frac{1}{4}}$, &c. Therefore the Fluxion of any ſurd Quantity is univerſally expreſſed as before, *viz.* the Fluxion of $y\frac{n}{m}$ is $\frac{n}{m}y\frac{m-1}{m}\dot{y}$, or $\frac{n}{m}y\frac{m-n}{n}\dot{y}$.

The Fluxion of the Logarithm of any Number or Quantity, is equal to the Fluxion of the ſaid Number, divided by the Number itſelf.

Let the Quantity $1+x$ be propoſed; the Fluxion thereof is $\dot{x}$, therefore $\frac{\dot{x}}{1+x}$ is the Fluxion of its Logarithm. For the Logarithm of $1+x$ is this, $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$, &c. the Fluxion of which is $\dot{x}-x\dot{x}+x^2\dot{x}-x^3\dot{x}$, &c. But that is equal to $\dot{x}\times\overline{1-x+xx-x^3}=\dot{x}\times\frac{1}{x+1}=\frac{\dot{x}}{1+x}$ as before. Thus the Fluxion of the Logarithm of $xx+yy$ is $\frac{2x\dot{x}+2y\dot{y}}{xx+yy}$, and of the Logarithm of ax^2+x^3 is $\frac{2ax\dot{x}+3xx^2\dot{x}}{ax^2+x^3}=\frac{2a\dot{x}+3x\dot{x}}{ax+x^2}$, &c.

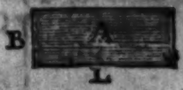
Note, That by the Reverſe of theſe Rules and Theorems, you find the fluent Quantities of any Fluxions given, tho' not with equal Certainty.

S E C T. XXVII.

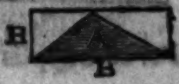
Of Menſuration of Superficies and Solids.

For a **Square**, put { S = its Side.
A = its Area.
Theor. $SS=A$.

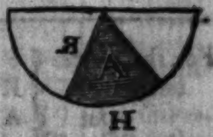
For a **Parallelogram**, put { L = The Length.
B = The Breadth.
A = The Area.
Theor. $LB=A$.



For a **Triangle**, put { H = Height.
B = Baſe.
A = Area.
Theor. $\frac{HB}{2}=A$.



For a **Sector** of a Circle. put { R = The Radius.
H = The Arch.
A = The Area.
Theor. $\frac{RH}{2}=A$.



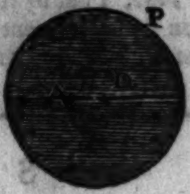
For a **Circle**, put { D = The Diameter.
P = The Periphery.
A = The Area. } Then any one of theſe being given, the other are thus found.

Theor. I. $3.1416D = P$.

Theor. II. $0.7854\,DD = A$.

Theor. III. $\left\{ \frac{P}{3.1416} = D \right.$. or $0.3183P = D$.

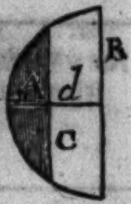
Theor. IV. $\left\{ \frac{PP}{12.5664} = A \right.$. or $0.07957\,PP = A$.



Theor. V. $\left\{ D = \sqrt{\frac{A}{0.7854}} \right.$. or $D = \sqrt{1.2732\,A}$.

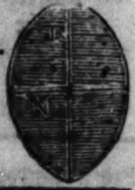
Theor. VI. $\left\{ \frac{A}{0.07957} = PP \right.$. or $P = \sqrt{12.5664\,A}$.

For the **Segment** of a Circle; put
R = The Radius.
d = Difference of the Ver. Sine and Radius.
C = Half the Chord of the Segment's Baſe.
A = The Area.



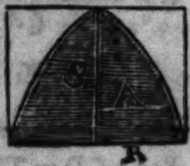
Theor. $\left\{ \frac{2\frac{1}{3}RR - 1\frac{1}{3}Rd - dd}{1\frac{1}{2}R + d} \times C = A \right.$.

For an **Ellipſis,** put
T = Tranſverſe Diameter.
N = Conjugate Diameter.
A = Area.



Theor. $0.7854\,TN = A$.

For the **Parabola,** put
S = Abſciſſa.
R = Semi-Ordinate.
A = Area.

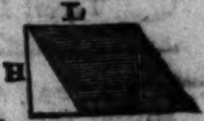


Theor. $\left\{ \frac{2SR}{3} = A \right.$.

For a **Spherical Triangle,** Let
A, B, C be the three Angles.
S = Superficies of the whole Sphere.
T = Area of the Triangle.

Theor. $\left\{ \frac{A+B+C-180^\circ}{720} \times S = T \right.$

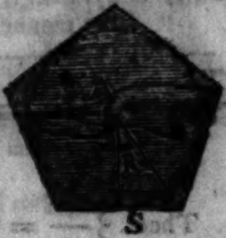
For a **Rhombus,** put
L = Length.
H = Height.
A = Area.
Theor. HL = A.



N.B. In the ſame manner a Rhomboides is meaſured.

For a **Regular Polygon;** put
R = Radius of the inſcribed Circle.
S = The Side in Length.
N = The Number of Sides.

Theor. $\left\{ \frac{RSN}{2} = A = \text{the Area.} \right.$



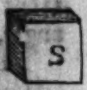
A Table for the more ready finding the Area of any regular Polygon.

Sides		
5	Pentagon,	1.720477
6	Hexagon,	2.598076
7	Heptagon,	3.633959
8	Octagon,	4.828427
9	Enneagon,	6.181827
10	Decagon,	7.694209
11	Endecagon,	8.514250
12	Dodecagon,	9.330125

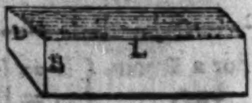
RULE. Multiply the Square of the Side by the tabular Number; the Product is the Area of the Polygon.

Of **Solids.**

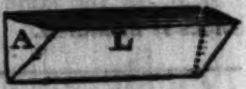
To meaſure a **Cube**, put
S = Its Sides.
Q = The Content or Solidity.
Theor. $SSS = Q$.



To meaſure a **Parallelopipedon**; put
L = The Length.
B = Breadth.
D = Depth.
Theor. $LBD = Q$.

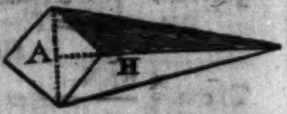


To meaſure a **Priſm**; put
A = The Area of its Baſe.
L = The Length.
Theor. $AL = Q$.

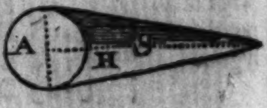


Note, A Cylinder is meaſured in the ſame manner.

To meaſure a **Pyramid**; put
A = The Area of the Baſe.
H = The Height.
Theor. $\frac{AH}{3} = Q$.



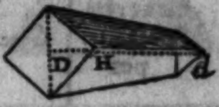
To meaſure a **Cone**; put
A = Area of the Baſe.
R = Radius thereof.
S = Side of the Cone.
H = its Height.



Theor. I. $\frac{AH}{3} = Q$.

And Theor. II. $\frac{SA}{R}$ = Curve Superficies.

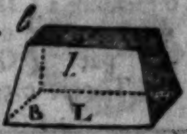
To meaſure the Fruſtrum of a Pyramid, Cone, &c. put
D = Side of the greater Baſe.
d = Side of the leſſer Baſe.
H = Perpendicular Height.
D—d = x.



Theor. $\overline{Dd+\frac{1}{3}xx}\times H=Q$.

Note, If it be the Fruſtrum of a right Cone, D and d will be the Diameter of the Baſes; and the Theorem for its Solidity will be $\frac{\overline{Dd+\frac{1}{3}xx}}{3.81968}\times H=Q$.

To meaſure a Priſmoid, put
L = Greateſt Length.
l = Leſſer Length.
B = Breadth of the greater Baſe.
b = Breadth of the leſſer Baſe.
$L+\frac{1}{2}l=x$, $\frac{1}{2}L+l=y$.
H = The Height.



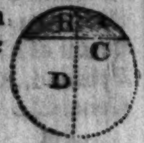
Theor. $\frac{\overline{Bx+by}}{3}\times H=Q$.

Note, If it be a Cylindroid; the Theor. is $\left\{\frac{\overline{Bx+by}}{3.81968}\right.\times H=Q$.

To meaſure a Sphere, or Globe; put
D = Diameter.
S = The Superficies.
Q = The Solidity.

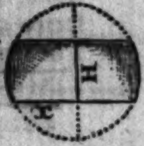
Theor. I. 3.1416 DD = S.
Theor. II. 0.5236 DDD = Q.

To meaſure the Segment of a Sphere; put
D = Sphere's Axis.
H = Segment's Height.
C = Half the Diameter of the Baſe.
Then we have two Theorems;



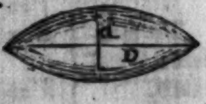
viz. Theor. I. $3CCH + HHH \times 0.5236 = Q.$
Or, Theor. II. $3DHH - 2H^3 \times 0.5236 = Q.$

To meaſure the middle Zone of a Sphere; put
D = The Diameter.
H = The Thickneſs.
x = The Diameter of the Baſe.

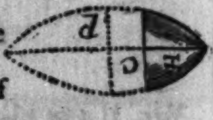


$$\text{Theor.}\left\{\frac{2DD + xx}{3.81968} \times H = Q.\right.$$

To meaſure a Spheroid; put
D = The longeſt Axis.
d = The Thickneſs.
Q = The Solidity.
Theor. $0.5236\, ddD = Q.$



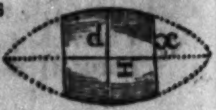
To meaſure the Segment of a Spheroid; put
d = The Thickneſs of the Spheroid.
H = The Height of the Segment.
c = Half Diameter of the Baſe.



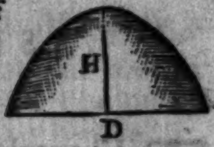
We have here alſo two Theorems;
viz. Theor. I. $3ccH + H^3 \times 0.5236 = Q.$
Or, Theor. II. $3dHH - 2H^3 \times 0.5236 = Q.$

The middle Zone of a Spheroid is meaſured as that of a Sphere, *viz.*

$$\text{Theor.}\left\{\frac{2dd + x^2}{3.81968} \times H = Q.\right.$$

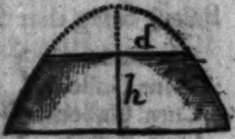


To meaſure a Parabolic **Conoid**; put
D = The Diameter of the Baſe.
H = The Height.
Q = The Solidity.
Theor. $0.3927\,DDH = Q$.

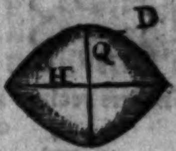


To meaſure the **Fruſtrum** of a Conoid; put
D = The Diameter of Baſe.
d = The Diameter of the part cut off.
h = The Height.

Theor. $\dfrac{DD+dd\times\frac{3}{2}h}{3.8196} = Q$.

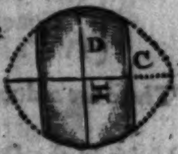


To meaſure a Parabolic **Spindle**; put
D = The Diameter or Thickneſs.
H = The Length, or Height.
Q = The Solidity.
Theor. $0.41888\,DDH = Q$.



To meaſure the middle **Zone** of a Parabolic Spindle; put
D = Diameter as before.
C = Diameter of the Baſes.
D − C = x.
H = The Height.

Theor. $\dfrac{2DD+CC-0.4xx\times H}{3.8196} = Q$.



N. B. The five regular **Bodies**, conſiſting of as many Pyramids, as they have Baſes; if the Content of thoſe Pyramids are found by the Theorem above, and added together, the Solidity of any of thoſe Bodies will be thence known.

SECT. XXVIII.

Of Gauging.

THE preceding Theorems for meaſuring Superficies and Solids, equally ſerve for finding the Capacity of any Veſſels; and therefore are the Foundation of Gauging. The Capacity of a Veſſel (conſidered as its Solidity) being thus found in cubic Inches, is reduced to

Ale Gallons,	by Dividing by	282	or Multiplying by	0.003546.
Wine Gallons,		231		0.004329.
Corn Gallons,		268.8		0.003722.
Corn Buſhels,		2150.42		0.000465.

In all circular Areas; only ſquare the Diameter, then

Divide by	359.05	or Multiply by	0.002785	For Ale Gallons.
	294.12		0.003399	For Wine Gallons.
	342.24		0.002922	For Corn Gallons.
	2738		0.0000036	For Corn Buſhels.

Gaugers reckon all cloſe Caſks to be in the Form of ſome one of theſe four Solids;

viz.
- I. The middle Zone or Fruſtrum of a Spheroid.
- II. The middle Zone or Fruſtrum of a parabolic Spindle.
- III. The lower Fruſtrums of two equal parabolic Conoids.
- IV. The lower Fruſtrums of two equal Cones.

Theſe are reduced to Cylinders, by finding a mean Diameter; thus

Multiply the Difference between the Head and Bung Diameters, by	0.7	for the	I.	Variety.
	0.65		II.	
	0.6		III.	
	0.55		IV.	

Then add the Product to the Head Diameter, and the Sum is the mean Diameter, or that of a Cylinder, whoſe Height and Content is equal, near as can be, to that of the Caſk.

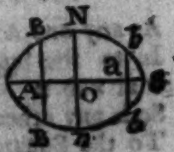
If you work by a Rule, the Gauge Points thereon are for

Square Areas,	or circular Areas	
16.79	18.95	For Ale Meaſure.
15.19	17.15	For Wine Meaſure.
46.36	52.32	For Malt Buſhel.

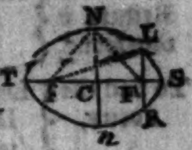
SECT. XXIX.

Of the **Conic Sections.**

The **Common** Property of all **Ellipſes.**



As the Rectangle of any two Abſciſſa's, TA × SA:
Is to the Square of half the Ordinate, *viz.* AB × AB ::
So is the Rectangle of any other two Abſciſſa's, Ta × Sa:
To the Square of half their Ordinate, *viz.* ab × ab.



To find the **Latus Rectum**; put
V = The tranſverſe Diameter TS.
G = The conjugate Diameter Nn.
L = The Latus Rectum, LR.

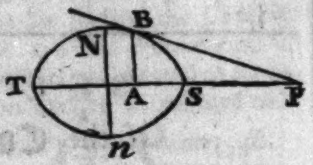
Theor. $\frac{GG}{V} = L.$

To find the **Focus**; put
V = The tranſverſe Diameter.
G = The conjugate Diameter.
FC = The Focal Diſtance.

Theor. $FC = \frac{1}{2}\sqrt{VV - GG}$.

Suppoſe CF = Cf, or F, f, the two Foci of the Ellipſis, then it will be FL + Lf = FN + Nf = TS. Hence the known Method of Drawing an Ellipſis mechanically.

To draw a **Tangent,** put $\begin{cases} D = TS. \\ X = AS. \\ z = AP. \end{cases}$



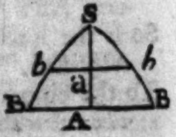
$$\text{Theor. I.} \left\{ \frac{DX - XX}{\frac{1}{2}D - X} = z. \right.$$

Theorem II.

$$\frac{D+z}{z} \pm \sqrt{\frac{DD+zz}{4}} = X.$$

The Common Property of all Parabola's.

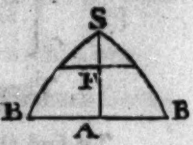
As any one Abſciſſa, Sa :
Is to the Square of its Semiordinate, ab × ab ::
So is any other Abſciſſa, SA :
To the Square of its Semiordinate, AB × AB.



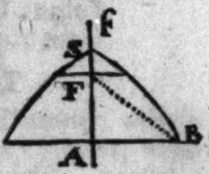
The Latus Rectum is thus found, { As any Abſciſſa : to its Semiordinate :: So is that Semiordinate : to the Latus Rectum. That is SA : AB :: AB : L = Lat. Rectum.

To find the Focus, { Let SF = be the focal Diſtance from the Vertex S.

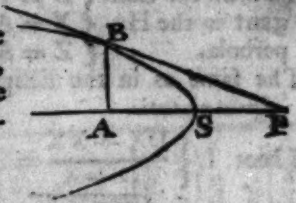
$$\text{Then} \left\{ \frac{AB \times AB}{4SA} = SF. \right.$$



Suppoſe F the Focus, and SF = Sf, then where-ever A be taken in the Axis; it will be SA + SF = FB, conſequently Af = FB. Thus as many Points B may be found as are ſufficient to form the Curve SB.

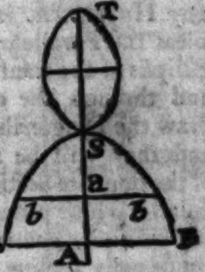


To draw a **Tangent** to the Curve of a Parabola, is very eaſy, for $AS = SP$, therefore B the Point of Contact is always determined.



The **Common** Property of **Hyperbola's.**

Let {
$X =$ Tranſverſe TS.
$Y =$ any Abſciſſa SA.
$Z =$ its Semiordinate AB.
$y =$ any other Abſciſſa aS.
$z =$ its Semiordinate ab.



Proportion $XY + YY : ZZ :: Xy + yy : zz$.

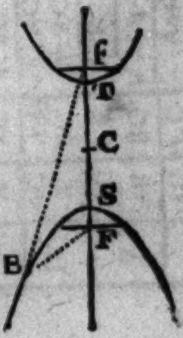
That is, $TA \times SA : BA \times BA :: Ta \times Sa : ba \times ba$.

But in the Ellipſis it is, $XY - YY : ZZ :: Xy - yy : zz$.

To find the **Latus Rectum.** { $XY + YY : ZZ :: X : L =$ Lat. Rect.

To find the **Focus**; put {
$F =$ The focal Point; then
$D =$ The focal Diſtance FS.
$B =$ Half the Tranſverſe $= \frac{1}{2}TS$.
$L =$ Latus Rectum.

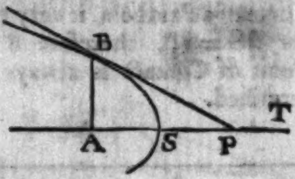
Then we have this Theor. $\sqrt{BB + \frac{1}{2}BL} - B = D = FS$.



To draw the **Curve**, { Suppoſe F and f the Foci of the two oppoſite Sections, then is $CF = Cf$; and it will be $fB - FB = TS$; or $fB - TS = FB$. And thus is it eaſy to deſcribe the Curve of any Hyperbola, by finding ſeveral Points B.

To draw the **Tangent** to the Hyperbola. $\left\{\begin{array}{l} D=TS. \\ X=AS. \\ Z=AP. \end{array}\right.$

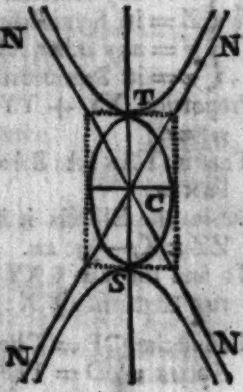
The ſame as in the Ellipſis, *ſignis mutatis.*



Theor. I. $\left\{ \frac{DX+XX}{\frac{1}{2}D+X}=Z. \right.$

Theor. II. $\left\{ \sqrt{\frac{DD+zz}{4}} \right.$
$+\frac{1}{2}Z-\frac{1}{2}D=X.$

To draw the **Aſymptotes** of any Hyperbola, is only to compleat the Parallelogram about the Ellipſis of the tranſverſe Diameter, and through the oppoſite Angles draw the Diagonals NCN, NCN, and they will be the Aſymptotes of the oppoſite Sections, or Hyperbola's; whoſe property is, that being infinitely continued, they will continually incline to the ſides of the Hyperbola, and yet never touch them.



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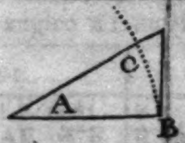
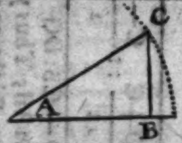
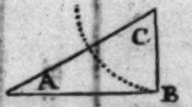
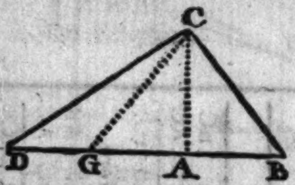
SECT. XXX. The Solution of Plain Triangles.

Of Right-angled Triangles, in all their Cases.

Cafes.	Given.	Sought.	 	 	
			Canons, or Analogies for Operation.		
I.	A AB	BC AC	R : AB :: tA : BC R : AB :: seA : AC	sC : AB :: sA : CB sC : AB :: R : AC	tC : AB :: R : BC tC : AB :: seC : AC
II.	AB BC	A AC	AB : R :: BC : tA $\sqrt{ABq+BCq}=AC$	Analogiâ caret AC as before.	CB : R :: AB : tC. AC as before.
III.	BC A	AB AC	A : BC :: R : AB tA : BC :: seA : AC	sA : BC :: sC : AB sA : BC :: R : AC	R : BC :: tC : AB R : BC :: seC : AC
IV.	AC A	AB BC	seA : AC :: R : AB. seA : AC :: tA : BC.	R : AC :: sC : AB. R : AC :: sA : BC	seC : AC :: R : CB. seC : AC :: tC : AB
V.	AC AB	A, C BC	AB : R :: AC : seA $\sqrt{ACq-ABq}=BC$	AC : R :: AB : sC BC as before.	Analogiâ caret BC as before.
VI.	AC BC	A, C AB	Analogiâ caret $\sqrt{ACq-BCq}=AB$	AC : R :: BC : sC. AB as before.	BC : R :: AC : seC. AB as afore.

Of Oblique Triangles, in all their Caſes.

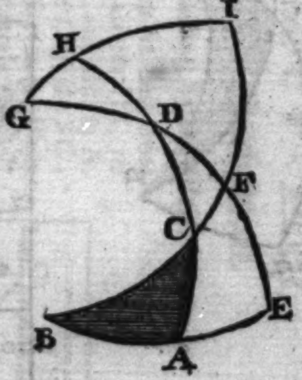


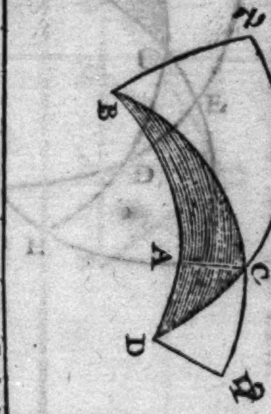
I.	CD, CB, D,	B	CB : sD :: CD : sB.
II.	CB, C, D,	DB	sD : CB :: sC : DB.
III.	CB. DB. B.	D, C,	$CB+BD : BD-CB :: t\frac{D+C}{2} : t\frac{C-D}{2}$
IV.	CD. CB. BD.	AB	BD : CB + CD :: CD — CB : DG Then DB—DG=GB, =2 AB; Thus you have reſolv'd it into two Right-angled Triangles, ABC, ADC.

SECT.

SECT. XXXI. The Solution of Spherical Triangles.

Cases.	Given.	Sought.	Of Right-angled Triangles in all Cases.	
			CANONS.	ANALOGIES.
I.	AB B	AC BC C	sBE : sBA :: tFE : tAC. sDE : sDF :: tAE : tCF.	R : sBA :: tB : tAC. R : csB :: ctAB : ctBC. R : csAB :: sB : csC.
II.	AC B	AB BC C	tEF : tAC :: sBE : sAB. sEF : sAC :: sBF : sBC. sCD : sCH :: sDF : sHI.	tB : tAC :: R : sAB. sB : sAC :: R : sBC. csAC : R :: csB : sC.
III.	BC B	AB AC C	sDF : sDE :: tCF : tAE sBF : sBC :: sEF : sAC sCF : sCI :: tDF : tHI.	csB : R :: ctBC : ctAB R : sBC :: sB : sAC. csBC : R :: ctB : tC.
IV.	AB AC	BC. B. C.	sAD : sDC :: sAE : sCF. sAB : sBE :: tAC : tEF	R : csCA :: csAB : csBC. sAB : R :: tAC : tB sAC : R :: tAB : tC.
V.	BC AC	AB B C	sDC : sDA :: sCF : sAE. sBC : sBF :: sAC : sEF. tIF : tDH :: sGI : sGH	csAC : R :: csBC : csAB. sBC : R :: sAC : sB. tBC : R :: tAC : csC
VI.	B C	AB AC BC	. . . . sHI : sDF :: sCH : sCD. tHI : tDF :: sCI : sCF.	sB : csC :: R : csAB. sC : csB :: R : csAC. tC : ctB :: R : csBC.





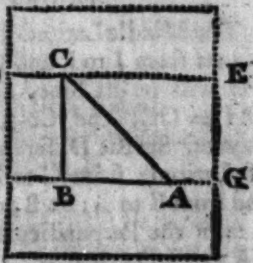
Cases.	Given.	Sought.	Of Oblique-angled Triangles in all Cases.	
I.	B. D. BC.	CD C BD	. . . . csBC : R :: ctB : tBCA R : csB :: tBC : tAB	D : sBC :: sB : sCD. sB : sBCA :: csD : sDCA. D : tB :: sAB : sAD.
II.	B. C. BC.	D DC BD	R : csBC :: tB : ctBCA R : csBC :: tB : ctBCA R : csBC :: tC : ctCBℵ	sBCA : sDCA :: csB : csD. sDCA : csBCA :: tBC : tDC. csCBℵ : csℵBD :: ctBC : ctBD
III.	BC. CD. B.	BD. C. D.	R : csB :: tBC : tAB R : csBC :: tB : ctBCA	csBC : csBA :: csDC : csDA tDC : tBC :: csBCA : csDCA. sDC : sB :: sBC : sD
IV.	BC BD B	CD D C	R : csB :: tBC : tBA. R : csB :: tBC : tBA. R : csB :: tBD : tBq	csBA : csBC :: csDA : csDC. DA : sBA :: tB : tD. Cq : sBq :: tB : tC.
V.	BC BD CD	C.	$sBC \times sCD : Rq :: s\frac{BD + AM}{2} \times s\frac{BD - AM}{2} : sq\frac{1}{2}C.$ *Note*, here AM = BC — CD. And q = Square. Then the Angle C being known, the other Parts are eaſy found.	
VI.	B C D	BD.	Change the Angles into Sides, (taking the Complement of the greateſt Angle C,) and this Caſe will then be the ſame with the laſt foregoing. For the Sides of this Triangle will be equal to the Angles of that.	

SECT. XXXII.

Of Navigation.

Sailing by the Plain Chart.

SUppoſe a Ship ſet Sail from C, in the Latitude DE, on the Rhumb AC, and ſails to A, which is in the Latitude FG; then 'tis evident that in the Plain Right-angled Triangle ABC,



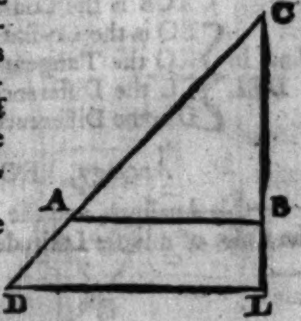
The Side
- CB is the Difference of Latitude.
- AC is the Diſtance ſailed.
- BA the Departure from the Meridian.

The Angle BCA is the Courſe of the Ship, or Angle made with the Meridian departed from.

Then if any two of theſe four Parts be known, the other two are found by the Analogies for ſolving Right-angled plain Triangles; Pag. 49. and herein is all the Art and Myſtery of Sailing by the Plain Chart.

Sailing by Mercator's Chart.

Mercator's Chart is a Correction and Improvement of the Plain Chart; but all its Propoſitions are ſolved by the Analogies of a Right-angled plain Triangle; though here we make uſe of two, *viz.* ABC, and DLC. The Parts of which each doth conſiſt, are theſe, *viz.*

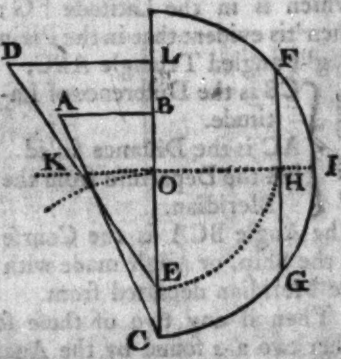


In ABC
- BC is the proper Difference of Latitude.
- AC is the Diſtance run.
- AB is the Departure.
- ACB is the Courſe.

In DLC { CL is the Meridional Difference of Latitude.
DL Difference of Longitude.

In either Triangle any two of thoſe Parts being known, the reſt are diſcover'd, as being but the ſeveral Caſes of a Right-angled plain Triangle, before reſolved. Pag. 49.

Sailing by Middle Latitude.



The Middle Latitude being ſet from I to F and G, FH will be its Sine, and OH = OE, the Co-ſine thereof. Set the Difference of Latitude from C to B, and from E to L; on B and L raiſe the Perpendiculars BA and LD; then according to what you have given, compleat the Triangle ABC; then from E, thro' the Point K, draw ED, to meet LD in the Point D. Thus you have conſtituted two Right-angled Triangles, ABC and DLE; and the Parts of which each doth conſiſt, are as follows; *viz.*

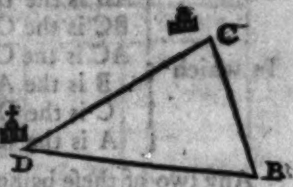
In ABC { CB is the Difference of Latitude.
CA is the Diſtance ſailed.
AB is the Departure;
ACB is the Courſe, the Ship ſteers.

And in DEL { EO is the Co-ſine of the middle Latitude.
KO the Tangent of the Courſe.
EL the Difference of Latitude.
DL the Difference of Longitude.

Analogy. EO : OK :: EL : LD.

By this Analogy, and the Triangle aforeſaid, ABC, all the Caſes of middle Latitude Sailing are ſolved.

Of Oblique Sailing.



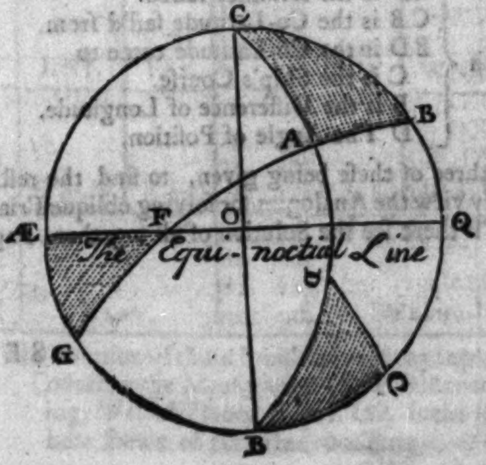
Suppoſe D and C, two Ports, &c. from which two Ships ſet ſail, and meet in B; then there will be form'd an Oblique Triangle DCB;

In which
DC = Diſtance of the Ports, &c.
CB = The Diſtance one Ship ſail'd.
DB = The Diſtance the other ſail'd.
C = The Angle of the firſt Ship's Courſe.
D = The Angle of the ſecond Ship's Courſe.
B = The Angle of Poſition, ſhewing how each Port bears from the Place of meeting B.

Now 'tis manifeſt, that if any three of theſe Particulars be given, the other may be found by the Analogies for the Solution of Oblique plain Triangles aforegoing. Pag. 50.

Orthodromiques, or Great Circle Sailing.

Suppoſe a Ship ſails from a Port B, on the Arch of a great Circle BG, to the Port A; then there will be form'd the Right-angled ſpherical Triangle ABC, right-angled at A.



In which
- AB is the Diſtance ſailed.
- BC is the Co-Latitude ſailed from.
- AC is the Co-Latitude arrived to.
- B is the Angle of the Ship's Courſe.
- C is the Difference of Longitude.
- A is the Right Angle.

Any two of theſe being given, the other three found by the Analogies for the Solution of Right-angled Triangles. Pag. 51.

Again, ſuppoſe a Ship ſails from G, to the Port F, in the Equator; then

In the Spherical Triangle GÆF
- G F The Diſtance ſailed.
- ÆF The Difference of Longitude.
- ÆG The Latitude.
- G The Angle of the Courſe.
- F The Angle of Poſition.
- Æ being the Right Angle.

Two of theſe being known, the reſt are known of courſe by the Analogies aforeſaid. Pag. 51.

But oblique ſpherical Triangles are moſt frequently made in Orthodromiques; therefore, ſuppoſe a Ship ſails from C to D; then there will be form'd the oblique Triangle BCD;

In which
- CD is the Diſtance ſailed.
- CB is the Co-Latitude ſail'd from.
- BD is the Co-Latitude come to.
- C is the Ship's Courſe.
- B is the Difference of Longitude.
- D The Angle of Poſition.

Any three of theſe being given, to find the reſt; you need only view the Analogies for ſolving oblique Triangles; and you'll there ſee the Solution of all together. Pag. 52.

SECT. XXXIII. A Table of Rhumbs.

	Rhumbs.		Meridian Angles.		Rhumbs.	
	North	South	°	′	South	North.
¼			2	49		
½			5	38		
¾			8	26		
1	N *by* E	S *by* E	11	15	S. *by* W	N *by* W.
¼			14	4		
½			16	53		
¾			19	41		
2	NNE	SSE	22	30	SSW	NNW.
¼			25	19		
½			28	8		
¾			30	56		
3	NE *by* N	SE *by* S	33	45	SW *by* S	NW *by* N.
¼			36	34		
½			39	23		
¾			42	11		
4	NE	SE	45	00	SW	NW.
¼			47	49		
½			50	37		
¾			53	26		
5	NE *by* E	SE *by* E	56	14	SW *by* W	NW *by* W
¼			59	4		
½			61	52		
¾			64	41		
6	ENE	ESE	67	30	WSW	WNW.
¼			70	19		
½			73	7		
¾			75	56		
7	E *by* N	E *by* S	78	45	W *by* S	W *by* N.
¼			81	34		
½			84	22		
¾			87	11		
8	East.		90	00	West.	

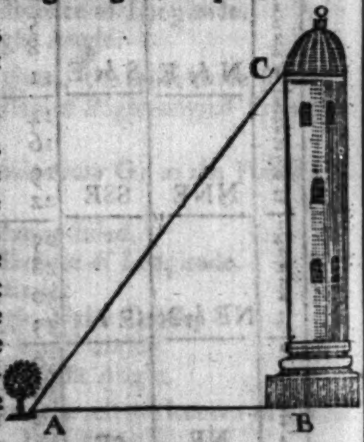
N.B. The Solution of thoſe Problems of Geography which relate to the Meaſuration of the Diſtances, Bearing, &c. of Places, Cities, &c. is the ſame as is here ſhewn of circular Sailing.

SECT. XXXIV.

Of **Altimetry** and **Longimetry.**

Altimetry, or Meaſuring Heights, Depths, *&c.*

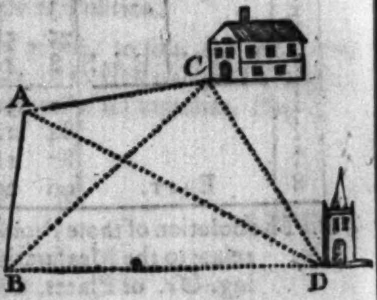
IN the Scheme adjoin'd, a Perſon at the Tree at A, taking with a Quadrant the Height of the Tower, *&c.* BC; or a Perſon on the Top of the Tower at C, taking the Depth to the Botom B, or taking the Angle of the Diſtance of the Tree at A; in either of theſe Caſes, 'tis very plain there will be form'd the Right-angled plain Triangle ABC.



In which { AB is the Diſtance of the Tower and Tree.
BC is the Height of the Tower.
A is the Angle of Altitude.
C is the Angle of the Diſtance of the Tree from the Tower; ſo that if any two of theſe be given, the other are known by the foregoing Analogies of Right-angled plain Triangles. Pag. 49.

Longimetry, of Menſuration of Diſtances.

Suppoſe a Perſon at A would meaſure the Diſtance between the Houſe C, and Spire D; firſt, let a Sight be taken of each at A, and then meaſure a Diſtance AB, and at the ſecond Station B, take a Sight of each again; then there will be form'd the following oblique Triangles.



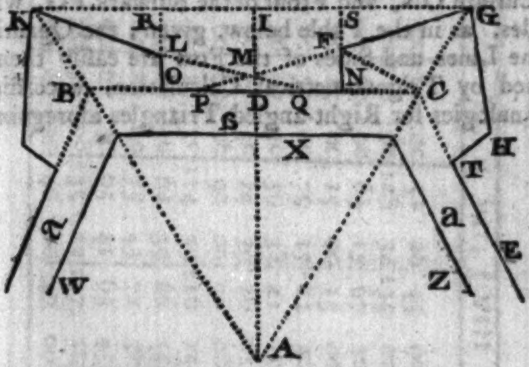
Viz. ABC, in which AB, and the Angles ABC and BAC are known; hence AC and BC, the Diſtances of the Houſe from A and B, will be known.

In the ſame manner in the Triangle ABD, the Sides AD and BD become known, which are the Diſtances of the Spire D from A and B.

Again, in either Triangle DBC, or DAC, there are two Sides DB, CB, or DA, CA, and the included Angle B or A, all given; by which DC, the Diſtance of the Houſe and Spire is eaſily known, by the Analogies for the Solution of Oblique-angled plain Triangles. Pag. 50.

SECT. XXXV.

Of **Fortification.**



A Deſcription of the

WXZ is a Plot of Ground, fortified in the Form of a Pentagon.
aaa The Rampire, or Wall of Earth.
ON The Curtain.
NFGHT A Bulwark.
NF, or OL, A Flank.
NT The Gorge of a Bulwark.
NC The Gorge Line.
CG The Head Line.
F, or L, the Shoulder.

ſeveral Lines and Angles.

K, or G, the Diamond Point, or flanked Angle.
FG the Face or Front of the Baſtion or Bulwark.
CG the Capital, or Head Line.
GO the longeſt Line of Defence.
PG or KQ, is the Line Stringent.
OP, or QN, is the ſecond Flank.
AD the ſhorteſt Line from the Center to the Curtain.

Angles.

BAC the Angle of the Center of the Polygon.
B, or C, the Angle of the Polygon.
FGH the flanked Angle, or Angle of the Baſtion.
NFG the Angle of the Eſpaul, or Shoulder.
SGF the Inward flanking Angle.
KMG the Outward flanking Angle.
CPF the Angle forming the Flank.

The Curtain ON, and Front of the Bulwark FG, with the Angles, as in the Table below, given; the Quantity of all the Lines and Sides of the Fort are eaſily thence determined by Trigonometrical Calculation, according to the Analogies for Right-angled Triangles aforegoing. Pag. 49.

The

A Table of the Dimenſions of the Angles obſerved in fortifying the regular Polygons following; *viz.* Pentagon, Hexagon, Heptagon, Octagon, Enneagon, and Decagon.

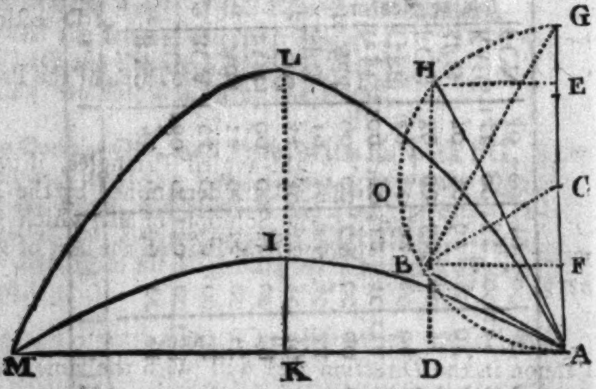
Note, there is no Neceſſity the Angles ſhould be juſt ſuch as are here ſpecified, but may be more or leſs at pleaſure.

The Number of Sides.	V.		VI.		VII.		VIII.		IX.		X.	
° ′												
BAC	72	00	60	00	51	26	45	00	40	00	36	00
BCE	180	00	120	00	128	34	135	00	140	30	144	00
BCA	54	00	60	00	64	17	67	30	70	00	72	00
Add	15	00	15	00	15	00	15	00	15	00	15	00
FGH	69	00	75	00	79	17	82	30	85	00	87	00
FGC	34	30	37	30	39	38	41	15	42	30	43	30
SFG	19	30	22	30	24	39	26	15	27	30	28	30
Add	90	00	90	00	90	00	90	00	90	00	90	00
NFG	109	30	112	30	114	39	116	15	117	30	118	30
GFC	59	30	62	30	64	39	66	15	67	30	68	30
FCG	86	00	80	00	75	43	72	30	70	00	68	00
SFG	70	30	67	30	65	41	63	45	62	30	61	30
KMG	141	00	135	00	135	22	127	30	127	00	123	00
FCN	40	00	40	00	40	00	40	00	40	00	40	00

SECT. XXXVI.

Of the Art of Gunnery, or Doctrine of Projectiles.

SUppoſe the Line AG repreſent the Force or Impetus, wherewith a Bullet, or other Body, is projected, and AB be the Direction; then will the parabolic Curve AIM be the Path of the Bullet's Motion, AM its horizontal Range or Diſtance; and $IK = AF$ the greateſt Height to which it will aſcend.



Deſcribe the Semicircle GHBA, and draw the Perpendicular BF, and the Lines BC, BG; and drop the Perpendicular BD; then we have $AD = \frac{1}{4}AM$; $AF = BD = IK$; $AC = CG = BC$; alſo the Angle $BAD = BGA$. Hence $BCA (= 2BGA) = 2BAD$. Laſtly, the Times or Durations of Projections are to one another, as the Lines of Direction contain'd within the Semicircle, or as the Chords of the Arches of Double the Elevation; that is, as the Sines of the Angles of Elevation. Theſe things premiſed, 'tis plain, that in order to ſolve all the Problems concerning the Impetus, Amplitude or horizontal Range, Height of the Random, Elevation or Angle of Direction, and the Duration of the Projection, *&c.* we have formed the Right-angled Triangles following;

Viz.

Viz. BAD In which	$AD = \frac{1}{4}AM$, or whole Random. $BD =$ The Height of the Random $= IK$. $AB =$ The Ratio of the Time of the Projection. $BAD =$ The Angle of Elevation or Direction.
Alſo, BCF In which	$BC = CA =$ The half Impetus, or Radius. $BF = AD = \frac{1}{4}AM$ or Random, as before. $BCF = 2BAD$ or double Angle of Elevation. $CF =$ The Difference of Radius, and Height $AF = KI$.
And ABG In which	$GA =$ The whole Impetus or Radius. $BGA = BAD =$ The Angle of Direction or Elevation. $AB =$ The Ratio of the Time of the Projection.

In any one of thoſe Right-angled Triangles, if two of the Parts be known, the reſt are determined by the Analogies proper thereto; the Solution of all which Caſes make up all the trigonometrical Part of the Art of Gunnery.

Note, If you make $CE = CF$, and draw HE; then a Cannon in the Direction AH will (with the ſame Charge AG) throw a Ball to the ſame Diſtance AM, as in the Direction AB; but its Path will be the Curve ALM, its Height LK, and the Time of its Duration to that of the former, as AH to AB; or as the Sines of the reſpective Angles of Elevation.

Note, 2. Becauſe CO is the greateſt of all Perpendiculars, as being $= AC$, or Radius; therefore four times $CO =$ to the greateſt Random, and which conſequently is made upon an Elevation of 45 Degrees.

SECT. XXXVII.

Of the Motion and Velocities of moving Bodies.

Suppoſe the Body . . . . . A
paſs over the Space . . . . S
in the Time . . . . T
And the Body . . . . . . B
paſs over the Space . . . . s
in the Time . . . . t

Call their Diſtance or Interval of Places . . D
And the Interval of the Times in which they begin to move . . I.

If A be fartheſt diſtant from the Place they tend to, or meet in, call that Diſtance X. Then there will ariſe two Caſes.

Case I. Suppoſe both Bodies tend the ſame way, or to the ſame Place,

Then if B moves firſt, we have this } Theor. I. $\left\{ X = \frac{SDt + SsI}{St - Ts} \right.$

But if A moves firſt, we ſhall have this . . } Theor. II. $\left\{ X = \frac{SDt - SsI}{St - Ts} \right.$

If they move both together, then $I = 0$; and conſequently . . . . . } Theor. III. $\left\{ X = \frac{SDt}{St - Ts} \right.$

Case II. Suppoſe they both move a contrary way, or meet;

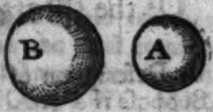
Then if B moves firſt, we have } Theor. IV. $\left\{ X = \frac{SDt - SsI}{St + Ts} \right.$

But if A moves firſt, we ſhall have . . } Theor. V. $\left\{ X = \frac{SDt + SsI}{St + Ts} \right.$

If they both move together, then $I = 0$, as before; and it is . . } Theor. VI. $\left\{ X = \frac{SDt}{St + Ts}. \right.$

For

For the Celerity of Bodies after Impact.



Suppoſe the Velocity of the moving Body A, be a; and the Velocity of B, be b. Let A be the ſwifteſt Body, and follow after (or meet) the Body B; and put X for the Velocity of A after the Impact; and Z for the Velocity of B; then here will be alſo two Caſes.

Case I. Suppoſe they tend both one way, we have theſe Theorems;

$$\text{Theor. I.}\left\{ X=\frac{Aa-Ba+2Bb}{A+B}\right.$$

$$\text{Theor. II.}\left\{ Z=\frac{2Aa-Ab+Bb}{A+B}\right.$$

If the Body B be at reſt, then will $b=0$; ſo that then we have

$$\text{Theor. III.}\left\{ X=\frac{Aa-Ba}{A+B}\right.$$

$$\text{And Theor. IV.}\left\{ Z=\frac{2Aa}{A+B}.\right.$$

Case II. If the Bodies tend the contrary way, or meet; we have

$$\text{Theor. V.}\left\{ X=\frac{Aa-Ba-2Bb}{A+B}\right.$$

$$\text{And Theor. VI.}\left\{ Z=\frac{2Aa+Ab-Bb}{A+B}\right.$$

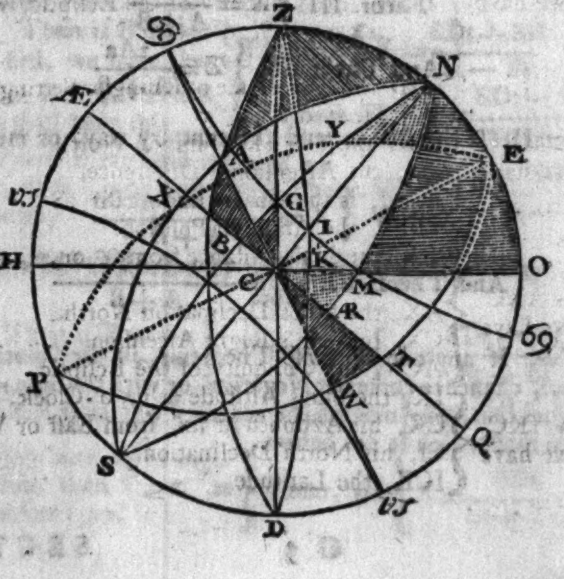
Note, If any one of thoſe Theorems, in the Solution thereof, chance to come out Negative, or with the Sign —, it argues that Body moveth the contrary way after Reflection to that which it did before.

SECT. XXXVIII.

Of Aſtronomy.

A Deſcription of the Lines and Circles in the adjacent Scheme.

ÆNQS, is the general Meridian.
NS, the Axis of the Sphere.
ÆQ, the Equinoctial Line.
HO, the Horizon of *London*.
♋C♑, the Ecliptic.
ZD, the Prime Vertical, or Azimuth of Eaſt and Weſt.
EP, the Axis of the Ecliptic.
NAS, NBS, NRS, Meridians, or Hour Circles.
ZAD, an Azimuth Circle.
EWP, A Circle of Longitude from the Ecliptic.
♋I♋, the Tropic of *Cancer*.
♑♑, the Tropic of *Capricorn*.



This Scheme repreſents the Projection of the Sphere or Globe rectified to the Latitude of *London*; and the ſeveral Lines abovemention'd, by their various Interſections, form various ſpherical Triangles, and thoſe of them which are moſt notable; and contain the moſt common and uſeful Problems of Aſtronomy, I have here ſhaded; ſix of which are Right-angled, *viz.* NOM, TWC, MRC, AXC, IKC, CBG; and one is Oblique-angled, *viz.* AZN. The Parts contain'd in every Triangle, are as follows;

In NOM we have	NO = ÆZ = the Latitude of *London*, always 51° 32′. OM, the Amplitude of the Sun's Riſing or Setting from the North. NM, the Co-Declination of the Sun. ONM, the Angle of the Hour from Midnight. OMN, the Angle of the Sun's Poſition.
In TWC we have	TW, the North Latitude of a Star at T. CW, the Longitude thereof in the Ecliptic. CT, the Right Aſcenſion. TCW, the Obliquity of the Ecliptic, which is ever 23° 29′. T, the Vertical Angle.
In MRC we have	CM, the Amplitude of Riſing or Setting from the E or W. MR, the Sun's Declination North. CR, the Aſcenſional Difference. RCM, is the Co-Latitude, 38° 28′. CMR, the Angle of Poſition.
In AXC we have	CA, the Sun's Diſtance from ♈ or ♎, in the Ecliptic. AX, the Sun's Declination North. CX, the Sun's Right Aſcenſion. ACX, the Obliquity of the Ecliptic.
In IKC we have	IK, the Sun's Altitude at ſix o'-Clock. CK, his Azimuth at ſix, from Eaſt or Weſt. CI, his North Declination. ICK, the Latitude.

In CBG we have
GB, the Sun's Declination North.
GC, the Sun's Altitude when due Eaſt or Weſt.
BC, the Time or Hour from ſix when the Sun is E or W.
BCG, the Latitude.

In the Oblique Triangle AZN, we have the following Things;

viz.
ZN, the Co-Latitude of *London* 38° 28′.
AN, the Co-Declination of the Sun, or Star, at A.
AZ, the Co-Altitude of the Sun or Star.
ANZ, the Hour from Noon = ÆX.
AZN, the Azimuth from the North.

To theſe we may alſo add the Oblique Triangle, YNE;

In which
YE, the Co-Latitude of a Star at Y.
YN, the Star's Co-Declination.
NE = Æ♋, the Diſtance of the Poles of the World and the Ecliptic.
NEY, the Star's Lengitude in the Ecliptic.
ENY, the Hour from Midnight.

I could have pointed out more Triangles, but theſe are ſufficient for my Purpoſe, ſince in reſolving of theſe by the Analogies beforegoing, you reſolve twenty or thirty of the moſt curious and uſeful Problems of ſpherical Aſtronomy.

The Dimenſions, and other general Affections of the Heavenly Bodies, the Planets, take from the beſt Authors in the following ſhort Synopſis.

The Dimenſions, &c. of the Planets.	*Saturn.*	*Jupiter.*	*Mars.*	*Earth.*	*Venus.*	*Mercury.*
Diameters in Engliſh Miles.	67870	81155	4444	7964	7906	2460
Diſtances from the Sun.	777000000	424000000	123000000	81000000	59000000	32000000
	D. H. '	D. H. '	D. H. '	D. H. '	D. H. '	D. H. '
Periodical Revolutions.	10759:06:36	4332:12:20	686:23:27	365: 6:09	224:16:49	87:23:16
Diurnal Revolutions.	. . . .	0: 9:56	1.00:40	0:23:56	0:23:00	. . . .
M. Ecliptic Motions.	00°:02':00"	00°.04':59"	00°:31':27"	00°:59' 08"	01°:36'.08"	04°.05':32"
Inclination of their Orbs.	02 :30:00	01 :20:00	01 :52:00	00 :00:00	03 :24:00	06 :54:00
Excentricities.	54700	25050	141000	1690	517	7970
Propor. of Gravity at each.	943	529	. . . .	435	. . . .	. . . .
Proportion of Bulks.	621350	1064500	170	1000	985	30
Their Denſities.	94½	67	. . . .	400	. . . .	. . . .
Quantity of Matter.	92	33	. . . .	0.59	. . . .	. . . .
Propor. of Light and Heat.	11	37	40	100	200	700
Their Moons.	5	4	. . . .	1	. . . .	. . . .

The Ecliptic Limits or Boundaries of the Sun and Moon are as follow;

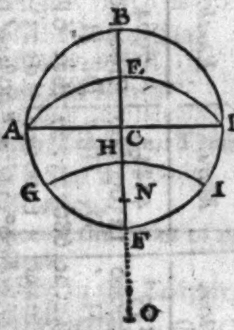
The greateſt Limits of	Moon.	12°:02':05"	Lat. 62'. 25".
	Sun.	18 :20:08	Lat. 94. 16.
The leaſt Limits of	Moon.	10°:19':17"	Lat. 53'. 41".
	Sun.	16 :35:05	Lat. 85. 32.

If the Luminaries are farther diſtant from the Nodes, in the new and full Moons, than the Limits here aſſign'd, there will be no Eclipſe; otherwiſe there muſt happen Eclipſes ſomewhere or other.

SECT. XXXIX.

The Stereographic Projection.

Prob. I. To draw an oblique Circle AED within the Primitive Circle ABDF, containning therewith the given Angle BDE = BE.

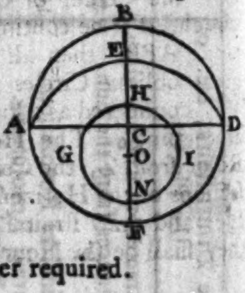


Rule. The Tangent of the given Angle, or BE, ſet off from the Center of the Primitive C, will give the Center N in the Diameter BE. Alſo the Secant of BE ſet off from E, will give the Center N, on which to deſcribe the great oblique Circle AED.

Prob. II. To deſcribe a leſſer Circle GHI parallel to the Right Circle ACD.

Rule. Set the Tangent of FI = FG, from H, and it will give, in the Diameter BF continued out, the Center o; or the Secant thereof ſet from C, will do the ſame; then on o you may deſcribe the Parallel required.

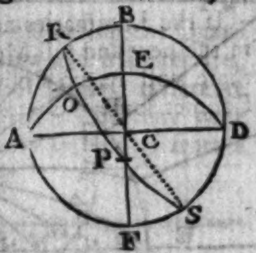
Prob. III. To draw a leſſer Circle GHIN parallel to an oblique Circle AED.



Rule. Set the half Tangent of its neareſt Diſtance from the Center C of the Primitive, to H; and the half Tangent of its greateſt Diſtance from the ſame, from C to N; then biſect the Diſtance HN in o, which gives the Center required.

Prob. IV. To deſcribe an oblique great Circle ROPS at Right Angles to a given oblique Circle AED.

Rule. Set the half Tangent of BE from C to P, ſo is P the Pole of the given oblique Circle AED. Then draw any Diameter RCS, and you have three Points R, P, S, through which the oblique Circle RPS being drawn, will interſect the given oblique Circle at Right Angles in o, as was required.



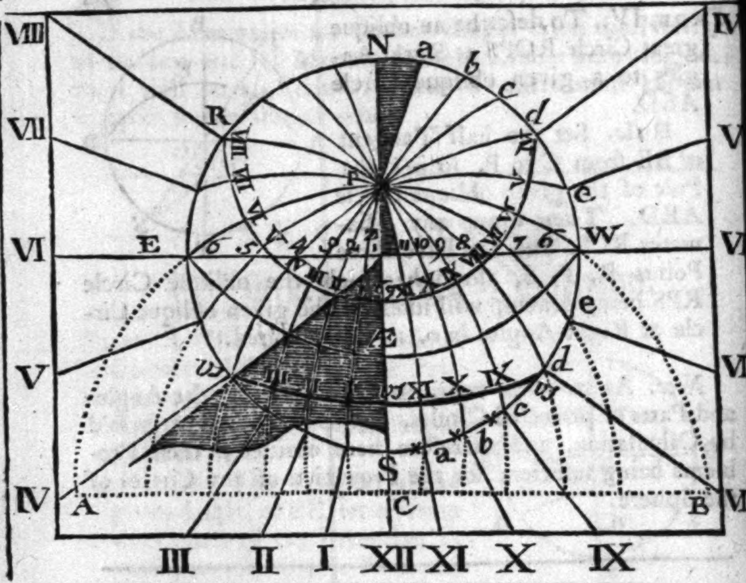
Note, As to what concerns the Meaſuring the Angles and Parts of projected Circles, that is much beſt perform'd by Calculation, and therefore here omitted; theſe Problems being ſufficient for the Projection of any Circles of the Sphere.

SECT. XL.

Of **Sciagraphy**, or the Art of **Dialling**.

I Have here adjoin'd the Scheme of an **Horizontal Dial**; with the Projection of the Sphere, to ſhew at ſight the Reaſon of the ſame.

The Circle ENWS repreſents the Horizon of *London*, within are projected all the Hour Circles, or Meridians; C is the Center of the Hour Circle of 6, *viz.* EPW, which, if it be continued out of the Projection to a Semicircle APB, then all the Hour Circles will appear to be great oblique Circles projected with the Hour Circle of VI, conſidered as a Primitive Circle, as taught in Prob. I. aforegoing. The Hour Circles thus projected, lay a Ruler from Z, the Center of the Projection, to the Interſections of the Horizon and Hour Circles, *viz.* S, a, b, c, *&c.* and draw ſtrait Lines on the Plane of the Dial, and they ſhall be the Hour Lines of XII, XI, X, IX, *&c.*



In order to find the Quantities of the Arches Sa, Sb, Sc, *&c.* by Calculation (without the Projection of the Sphere) the Triangles, NPa, NPb, NPc, *&c.* will enable you to find the Arch Na = Sa, Nb = Sb, *&c.* for therein is known the Side NP, = the Latitude 51° 32′, and the Angle NPa, = 15° or NPb = 30°, *&c.* and the Right Angle at N; to find Na, Nb, *&c.* which is done by Caſe I. of Right-angled ſpherical Triangles; and thus you not only find the Arches Sa, Sb, *&c.* anſwering

Hours.	D.	M.	Hours.	D.	M
XII.	00	00	IX. III.	38	03
1	2	56	1	41	45
2	5	52	2	45	34
3	8	51	3	49	30
XI. I.	11	51	VIII. IV.	53	35
1	14	52	1	57	47
2	17	57	2	62	06
3	21	06	3	66	33
X. II.	24	20	VII. V	71	06
1	27	36	1	75	45
2	31	00	2	80	2
3	34	28	3	85	13
			VI.	90	00

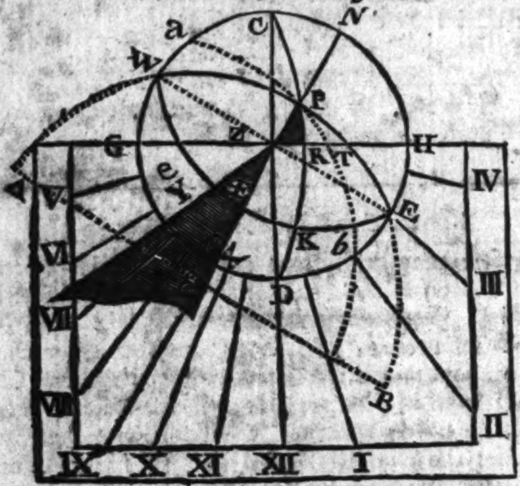
ſwering to the Hours, but alſo thoſe anſwering the Quarters. And all thoſe Arches in Degrees and Minutes are, for an Horizontal Dial in the Latitude of *London*, as here ſet down in the Table adjoin'd; by which a Dial may very ſpeedily be made, by ſetting thoſe Numbers off from a Line of Chords on any proper Circle.

For an Erect Direct South Dial.

In the foregoing Projection, the Diameter EZW repreſents the Section of the Plane of an Erect Direct South or North Dial; therefore if a Ruler were laid from N to the Points Z, 11, 10, 9, &c. where the Hour-Circles interſect it; and where the Ruler cuts the Primitive Circle were made ſmall Marks, ×××, &c. then a Ruler laid from Z to each of thoſe Marks, and Lines drawn thereby on the Plane of the Dial would be the true Hour-Lines required. Otherwiſe they are found by Calculation, thus; in the ſmall Right-angled Triangle ZP11, there is known the Side ZP = Co-Latitude of *London*, 38° 28′. and the Angle ZP11 = 15°, to find the Side Z11, which is done by Caſe I. (as before) of Right-angled ſpherical Triangles. Thus having found the Diſtances Z11, Z10, Z9, &c. they will be equal to the Arches S×, S×, &c. which being calculated for every Hour and Quarter will be found ſuch as in the Table adjoin'd you ſee them. Now, by means of this little Table, ſuch a Dial may be inſtantly made, without farther Trouble.

Hours.	D.	M.	Hours.	D.	M.
XII.	00	00	IX. III.	31	54
1	2	20	1	35	22
2	4	41	2	39	3
3	7	3	3	42	58
XI. I.	9	28	VIII. IV.	47	9
1	11	56	1	51	36
2	14	27	2	56	20
3	17	4	3	61	23
X. II.	19	45	VII. V.	66	42
1	22	35	1	72	17
2	25	32	2	78	3
3	28	38	3	80	0
			VI.	90	0

Of Vertical Declining Dials.



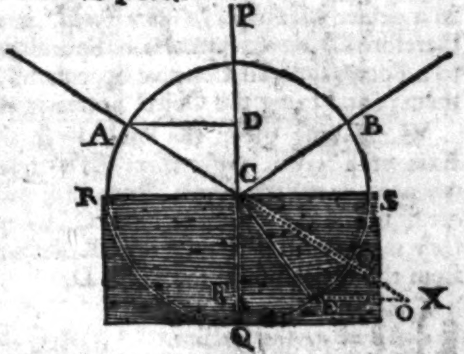
In order to make ſuch a Dial, deſcribe the Primitive Circle WNES, wherein let CD be the Meridian of the declining Plane, and ſuppoſe it declines 30° to the Eaſt; then make the Arch CN = 30°, and draw NS for the Axis of the Sphere, WPE is the Hour-Circle of Six paſſing through the Eaſt and Weſt Points E, W. WÆE is the Equinoctial; P, the Pole of the World; CPD, a great Circle, perpendicular to the Plane GH, paſſing through the Pole P. aTb is the Hour-Circle of Five; theſe Lines and Circles thus drawn, there will be conſtituted the Right-angled ſpherical Triangle ZPR.

In which
ZP = the Co-Latitude 38° 28′.
PR = XY, the Stile's Height.
ZR = DS, the Subſtile's Diſtance from the Meridian CD.
PZR = the Co-Declination = NH = 60°.
ZPR = ÆK, the Plane's Difference of Longitude.

Now, becauſe ZP and PZR, with the Right Angle at R, are always known; therefore the other three Requiſites are eaſy found by Caſe III. of Right-angled ſpherical Triangles beforegoing. Then to find the Hour Diſtances from the Subſtile, proceed thus; ſuppoſe the Diſtance of the Hour Line of V from the Subſtile be required; to find which, if from the whole Angle ZPT the Angle ZPR be ſubſtracted, there will remain the Angle RPT known, but there is alſo known PR, with the Right-Angle R, in the Triangle RPT; wherefore we can find RT = the Diſtance of the Hour Circle of V (*viz.* aTb) from the Subſtile R, which ſet from X to e in the Primitive, and from thence draw the Hour Line eV; and thus may all the other Hour Lines be drawn, and the Dial finiſhed as above.

SECT. XLI.

Of Optics.



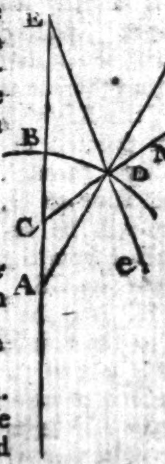
I. SUppoſe RS be a Surface of Water, Glaſs, *&c.* and AC a Ray of Light paſſing thro' the Air, and falling thereon at C; this Ray will not proceed directly through the ſame to O, but will be refracted from thence towards the Perpendicular CQ in the Direction CE.

II. And the Sine AD of the Angle of Incidence ACP, is to the Sine EF of the Angle of Refraction ECQ, always as 4 to 3; if out of Air into Water; but if the Refraction be made out of Air into Glaſs, the Sine of Incidence AD is to the Sine of Refraction EF as 17 to 11, (or more nearly) as 31 to 20, or as 77 to 50, for Rays of a red Colour.

III. For Sir *Iſaac Newton* has diſcover'd, that all Colours exiſt in the Rays of Light; and in this Order, *viz. Violet*, *Indigo*, *Blue*, *Green*, *Yellow*, *Orange* and *Red.* The Quantity of thoſe Colours in the Rays of Light are to one another in muſical Proportion, or as the Intervals of theſe Numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{10}$, $\frac{1}{2}$; now all theſe are the Intervals or Notes into which the Octave, or muſical Chord is divided, and the different Refrangibility of thoſe different coloured Rays, is according to thoſe Numbers 78, $77\frac{7}{9}$, $77\frac{2}{3}$, $77\frac{1}{2}$, $77\frac{1}{3}$, $77\frac{1}{5}$, $77\frac{1}{8}$, 77, the Intervals of theſe Numbers correſponding to the Sines of Refraction of the Rays producing the Colours in the Order beforementioned.

IV. The Angle of Incidence ACP is always equal to the Angle of Reflection PCB. Any Object at E will be viſible to an Eye at A, but will ſeem to be in X, in the Ray AX; thus by means of Water, Objects may be made Viſible, which otherwiſe are not ſo.

V. Becauſe the Refraction is towards the Perpendicular in a denſer Medium, *&* *vice verſâ*, in a rarer Medium; therefore all convex Surfaces, or Speculums, converge Rays to a Focus; and all Concave Speculums diverge or ſpread them; and bring the Object nearer to the Sight.



VI. Having given the Point E in the Axis of a given Speculum BD, from whence, or toward which, Rays (Ee) proceed, to find the Focus or Point C, where they meet again and unite after Reflection from the Speculum in the Point D.

put $\begin{cases} EB = d. \\ AB = r, \text{ the Radius.} \\ BC = x, \text{ the Focal Diſtance.} \end{cases}$

Then $AC : CB :: AE : EB$, that is $r - x : x :: d + r : d$; whence ariſes this Equation $dr - dx = dx + rx$; which gives $\frac{dr}{2d + r} = x = BC$, for a general Rule or Canon. This Canon (*mutatis mutandis*) is applicable to all poſſible Caſes both of *Convex* and *Concave Speculums*, and of *converging*, *diverging* and *parallel* Rays.

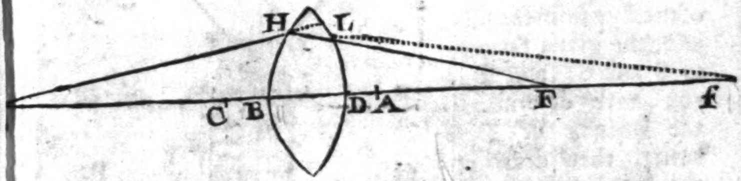
VII. To magnify or diminiſh a given Object by any given Speculum in any aſſign'd Proportion;

Let $\begin{cases} b \text{ repreſent the given Object.} \\ c \text{ the Image thereof to be form'd.} \\ d, \text{ and } r, \text{ as before.} \end{cases}$

Then it will be $b : c :: d : x$, and ſo $\frac{cd}{b} = x = \frac{dr}{2d + r}$; and conſequently $d = \frac{br - cr}{2c}$, the univerſal Canon for finding the Diſtance required for the Poſition of the given Object from the Speculum.

VIII. Let HBLD be a *Lens* of Glaſs, *&c.* EH, a Ray of Light falling thereon in H, through which it is refracted to L, and again from thence to the *Focus* F, in the Axis of the Lens.

$$\text{And put} \left\{ \begin{array}{ll} EB = d. & \\ AB = r, & \text{the Radius of HB.} \\ CD = \rho, & \text{the Radius of LD.} \\ BD = t, & \text{the Thickneſs of the Lens.} \\ Bf = x. & \\ Df = y. & \end{array} \right.$$



And alſo let I and R be the Sines (or expreſs the *Ratio* of the Sines) of Incidence and Refraction. Then $Bf = x = \frac{Idr}{Id - Rd - Rr}$, is the Theorem which ſerves univerſally for finding the Focus f after Refraction from the firſt Surface at H, call'd the *Focus of Tranſition.*

Now, in order to find the Focus F after the ſecond Refraction, if we put $p = \frac{R}{I - R}$ we ſhall have the following Theorem; $DF = y = \frac{Ipdr\rho - Rd\rho t + Rpr\rho t}{Idr - Idt + Rrt + Id\rho + Rdt - Ipr\rho}$.

If t, or the Thickneſs of the Lens be neglected, as it may, being inconſiderable, the Theorem will be thus abridg'd, *viz.* $DF = y = \frac{pdr\rho}{dr + d\rho - pr\rho}$.

This univerſal Theorem is a kind of Synopſis of *Dioptrics*, as finding the Focus of all kind of Lenſes, and all ſorts of Rays.

If you would magnify or diminiſh any Object in the Proportion of b to ϱ, the Diſtance or Poſition of the Object is $EB = d = \frac{2r\varrho + 2rb}{r+\varrho}$.

SECT. XLII.
Of **Perſpective.**
Of **Lineal** Perſpective.

I. LET ABCD be a Geometrical Square, whoſe perſpective Appearance is required. At the given Height of P the **Point** of **Sight** (= Sn) above the **Ground** Line nm, draw parallel thereto the **Horizontal** Line SPS; in which from P ſet the Diſtance of the Eye from the Object (the given Square) to S and S; and thus you have determined the **Points** of **Diſtance**, then draw up the **Radials** AP and BP; and draw the **Diagonals** AS, BS, and through the Points E, F, where theſe cut the Radial, draw the **Tranſverſe** EF; then is AEFB the Perſpective Appearance (in a direct View) of that given Square ABCD, in *Fig.* I. Hence the ſide of the Square CD is diminiſhed into EF, the ſide AC into AF, the Diagonals AD, CB, into AF, EB; and all the ſmall Squares, Parallelograms, and Triangles, form'd in the Geometrical Square appear in the Perſpective Square, as in the Schemes. *Note*, *Fig*. II. ſhews you the Perſpective of the ſame Geometrical Square, &c. in an oblique View.

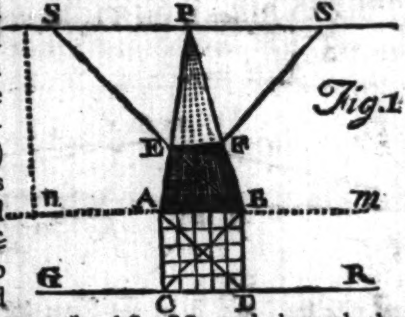


Fig. I.

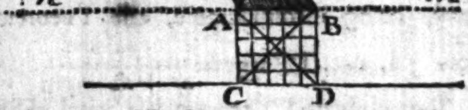
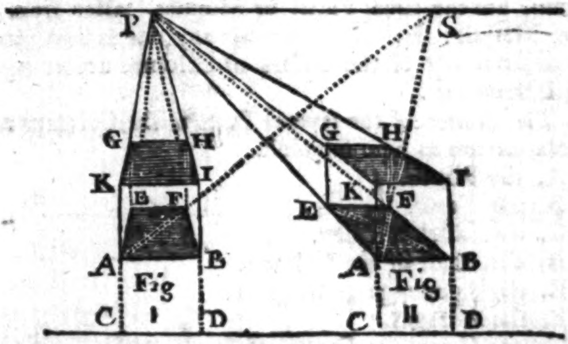


Fig. II.

II. Suppoſe it required to exhibit the Perſpective Appearance of a Cube or Die, whoſe Side is AKIB, the Point of Sight being P, and PS the Diſtance.

I. Make ABCD = AKIB, and compleat the Perſpective Square AEFB, as taught above.

II. From KI draw up the Radials KP, and IP; draw EG, and FH parallel to AK and BI, and where they cut the Radials in G, H, draw the Tranſverſe GH, and thus is the Cube compleated as required. For AEFB, and KGHI, are the lower and upper Faces thereof cloſe; GHFE, the hinder, and AKIB, the Fore-ſide or Front; and AKGE, BIHF the two oppoſite Sides, all open. *Fig.* I. is the direct View and *Fig.* II. the oblique View.



SECT. XLIII.

Of Architecture.

1. ARchitecture is the Art of Building or raiſing all kinds of Edifices; as, Houſes, Churches, Palaces, &c.

2. The principal Subjects of this Art, are the various Kinds or Orders of Columns or Pillars, and Pilaſters; with their different Proportions; and their ſeveral Members, Ornaments, &c. of all which, a little.

3. The moſt common and notable Orders of Columns, are the five following, *viz.* 1. The *Tuſcan*; 2. the *Doric*; 3. the *Ionic*; 4. the *Corinthian*; and 5. the *Compoſite* Order.

4. Of each of theſe I have given a curious Print, with the Proportions and Projectures of the ſeveral Members, expreſs'd in Numbers affixed thereto, according to that celebrated Architect, ANDREA PALLADIO.

5. The Columns ſupport the Entablature of Buildings; and both the Column and Entablature do conſiſt each of three great Parts, *viz.* The Column conſiſts of the *Baſe*, the *Fuſt* or *Shaft*, and the *Capital*. The Entablature, of the *Architrave*, *Freeze*, and *Corniſh*. See the Column of the *Compoſite* Order.

6. The Columns, and their various Members, as alſo thoſe of the Entablature, are all drawn and proportion'd by a Scale, call'd a *Module*, as AB; which is divided into 60 equal Parts, call'd Minutes. This Module is always equal to the Diameter of the Column at the Baſe; and the Numbers expreſſing the Proportions of the Parts in each Column, are the equal Parts, or Minutes, taken from its proper Module: ſo that 'tis eaſy for any one to underſtand how to form any of the Orders of Columns according to thoſe Proportions.

7. The Names of the ſeveral Parts of the Columns and Entablature are as here ſubjoin'd.

In the *Tuſcan*,
A, the Plinth.
B, the Torus.
C, the Liſtel or Fillet.
D, the Shaft of the Column.
E, the Apophyge or Cincture.
F, the Aſtragal.
G, the Gorge, or Freeze of the Capital.
H, the *Ovolo* or *Echinus*.
I, the *Abacus*.
K, the ſmaller } Face of the *Architrave*.
L, the greater } Face of the *Architrave*.
M, the *Liſt* of the *Architrave*.
N, the *Freeze* or *Zophoros*.
O, } the two Parts of the *Talon*.
P, } the two Parts of the *Talon*.
Q, the *Corona*, Crown, or *Larmier*.
R, the *Cima Recta*, or *Ogee*.

In the *Doric*,
A, the Scotia.
B, the *Guttæ*, or Drops.
C, C, the Triglyphs.
D, the Metops.

In

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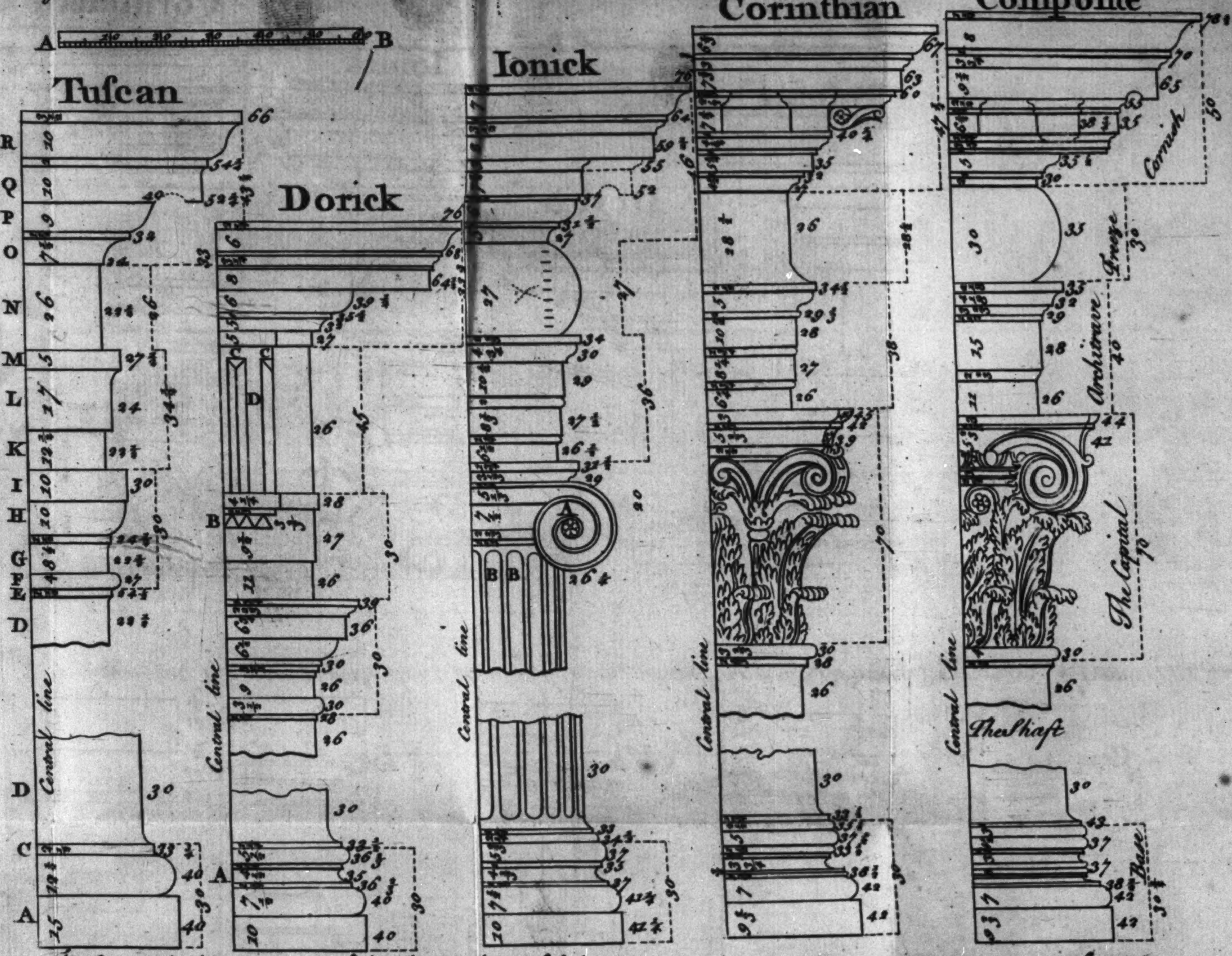

*The Geometrical Elevations of the five Orders of Columns According to y^e^* **Celebrated Proportions** *of* ANDREA PALLADIO.

In the
Ionic,
8. T
Compoſi
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and ſor

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II.

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The

In the *Ionic*, { A, the Volute, or Scroll.
B, B, the Flutings.

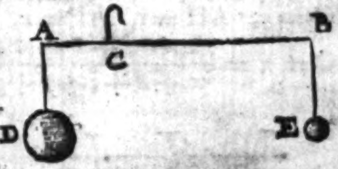
8. The Capital of the Column in the *Corinthian* and *Compoſite* Orders, is embelliſhed with curious Carvings of Thiſtle-Leaves, ſay ſome; of Olive-Leaves, ſay others; and ſome ſay they are of Palm, as you ſee in the Plate.

SECT. XLIV.

Of Mechanics.

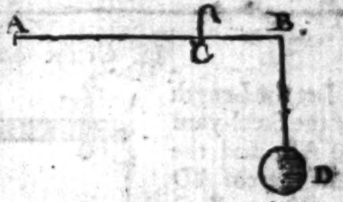
I. Of Balance.

I. { Suppoſe the Length of Balance, AB $= s$.
The ſhorter Brachium, or Arm, AC $= a$.
The greater Weight, D $= d$.
The leſſer Weight, E $= e$.



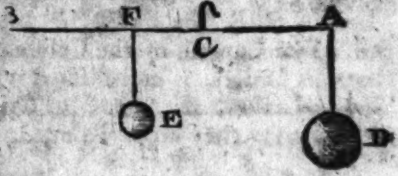
Then any three of theſe being given, the other may be found by the following Theorems; *viz.* Theor. I. $a = \frac{se}{d+e}$. Theor. II. $s = \frac{da}{e} + a$. Theor. III. $d = \frac{se}{a}$. IV. $e = \frac{da}{s-a}$.

II. { Let the Length of the Balance, AB $= s$.
The Weight of the Balance be $= w$.
The Diſtance CB $= a$.
And the Weight applied, D $= d$.



Then Theor. I. $a = \frac{sw}{d+w}$. Theor. II. $S = \frac{ad}{w} + a$.

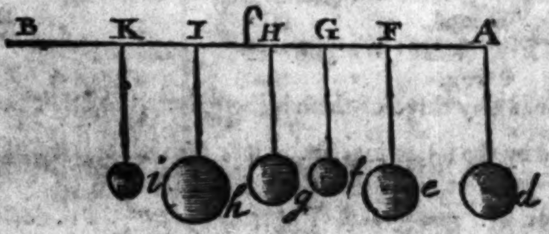
Theor. III. $w = \frac{ad}{s-a}$. IV. $d + \frac{sw}{a} - w$.



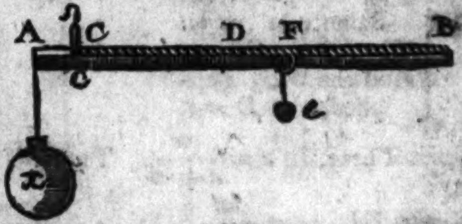
III. Again, let the Balance be AB $=2s$.
Alſo let the Weight thereof be $=w$.
Let the Weight hung at one End D $=d$.
And another in any Point F be E $=e$.
Let the Length . . . AF $=q$.
And the Diſtance . . . AC $=z$.

Then we have this Theorem $z=\frac{sw+eq}{w+d+e}$.

IV. And univerſally, for any Number of Weights d, e, f, g, h, i, &c. by putting their Diſtances AF $=q$, AG $=r$, AH $=t$, AI $=u$, AK $=x$, &c. we have AC $=z=\frac{sw+eq+fr+gt+hu+ix}{w+d+e+f+g+h+i}$, &c.



II. Of the Steel-Yard.

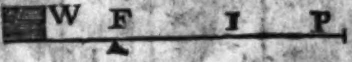


Let the Length of the Steel-yard be AB, and the half thereof AD $=$ DB, and C the Center of Motion; x the Weight to be weighed, e the ponderating Weight, and call the Weight of the Steel-yard itſelf w. Alſo put AC $=a$, and CD $=b$, and CF $= x$, the Diſtance to be found. And we have this Theorem, $z = \frac{ax - bw}{c}$.

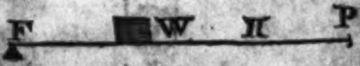
III. Of the Lever.

Of Levers, there be four Kinds; *viz.*

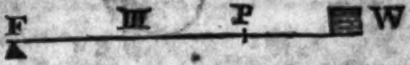
I. When the Weight W is at one End, the Power P applied to the other End, and the Fulcrum (or Prop) F is between them.



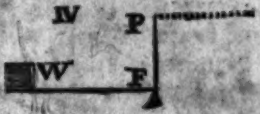
II. When the Fulcrum F is at one End, the Power P applied at the other; and the Weight W between.



III. When the Fulcrum F is at one End, the Weight W at the other; and the Power P applied between.



IV. When the Lever is crooked, making an Angle at the Fulcrum, as a Hammer in drawing a Nail. See the Figure.



Now in any of theſe Caſes, calling the Weight W, the Power P, and putting the Diſtance WF $= a$, and PF $= b$; we ſhall have, $p : w :: a : b$; and conſequently, $\frac{wa}{b} = p$. $\frac{pb}{a} = w$. $\frac{pb}{w} = a$. and $\frac{wa}{p} = b$.

IV. Of the Pulley.

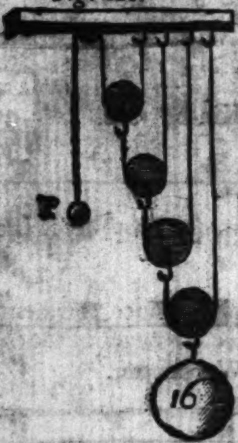


Fig. III.

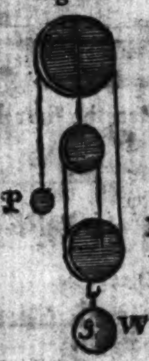


Fig. I.

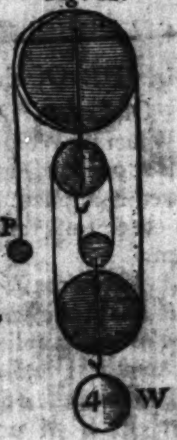


Fig. II.

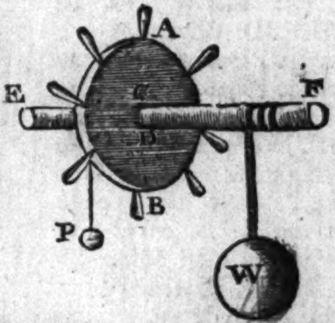
I. The Diſpoſition, or manner of fixing Pullies is various; if it be as in *Fig.* I. and II. then the Power P will be to the Weight W, it ſuſtaineth, as Unity or 1, to the Number of Ropes applied to the lower Pullies. Thus in *Fig.* I. if the Power P = 1 Pound, the Weight W = 3 Pounds, and in *Fig.* II. the Weight W = 4 Pounds, if P = 1 Pound.

II. But if (as in *Fig.* III.) they be ſo combined, as that each Pulley hath one of its Ropes fixed, and is attracted by the other, then each Pulley doubles the Force; and thus, if P = 1 Pound, it will equipoiſe two Pound by the firſt Pulley, 4 by the ſecond, 8 by the third, and 16 by the fourth Pulley; and ſo on if there were more.

V. Of

V. Of the Wheel and Axel.

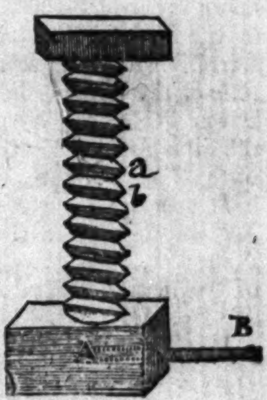
In the Figure adjoin'd, let AB be the Wheel, and EF its Axle; then the Power P will be to the Weight W, which it ſuſtains, as the Diameter CD of the Axle to the Diameter of the Wheel AB, or as the Circumference of the Axle to that of the Wheel.



Note, If the Wheel be turn'd by Handles or Spokes, then twice the Length of thoſe Handles muſt be added to the Diameter of the Wheel, in Computation.

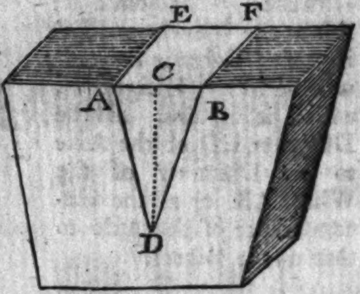
VI. Of the Screw.

In the Action of Screws, their Force or Power of Raiſing or Depreſſing is computed by this Analogy; *viz.* as the Diſtance a b of any two contiguous helical Threads, is to the Circumference deſcribed by the Power applied to a Lever (or Hand-Bar) at B in one Revolution, ſo is the Power to the Weight, Reſiſtance, or Force of the Screw.



Note, In this, and all other Machines, there muſt be an Allowance made for Friction, more or leſs.

VII. Of the Wedge.



Let the Wedge be AEFBD driven into a tough Piece of Wood, as in the Figure. Then, as the Thickneſs of the half Wedge AC is to the Height thereof CD, ſo is the Power of the Wedge to the Reſiſtance of the Wood on each Side.

Hence, if the Power be increaſed, the Reſiſtance will be overcome, and the Wood will be cleſt.

VIII. Of the Inclined Plane.

I. If a Weight W be ſuſtain'd on an inclined Plane RB (*Fig.* I.) by the Power P, and its Line of Direction RW be parallel to the Baſe of the Plane AB; then the Power P is to the Weight W, as the Sine AR to the Co-ſine AB of the Plane's Inclination ABR.

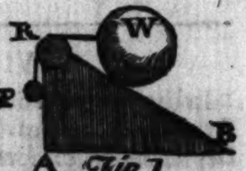


Fig 1

II. If the Line of Direction RW be parallel to the inclined Plane RB, then P : W :: the Sine of Inclination RA : Radius RB. See *Fig.* II.

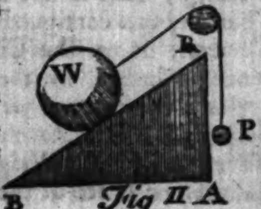


Fig II

III. If two Weights W, P, ſuſtain each other on two inclined Planes joined RB, and RA (*Fig.* III.) whoſe Line of Direction WP is parallel to the Horizon AB; then W : P :: BD : DA.

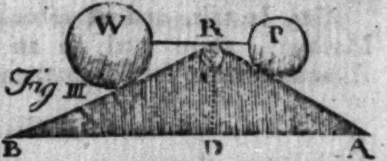
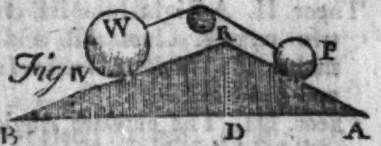
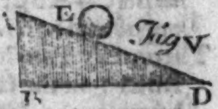


Fig III

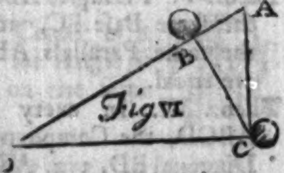
IV. If their Lines of Direction be parallel to the inclined Planes RB, RA; then is W : P :: RB : RA. *Fig.* IV.



V. The Velocity which a Body E will have when it hath paſſed through the inclined Plane AD at D, is to the Velocity it would acquire in falling through the Perpendicular to B in a Ratio of Equality; that is, the Velocity at B is equal to that at D. *Fig.* V.

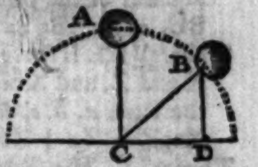


VI. In *Fig.* VI. If from the Right Angle C, you let ſall the Perpendicular CB to the inclined Plane AD; then a heavy Body will fall on the Plane to B, in the ſame time it would deſcend through the Perpendicular to C.



IX. Of Direct and Oblique Percuſſion.

If a Body impinge on the Point C, directly in the Perpendicular AC, and again obliquely in the Direction BC; the Celerities being the ſame in both, the Power of the Direct Stroke in AB, will be to the Power of the Oblique Stroke in BC, as Radius AC or BC is to the Sine BD of the Obliquity BCD.

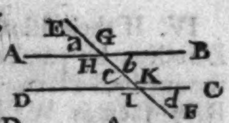


SECT. XLV.

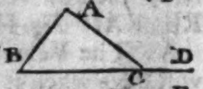
Of Geometry.

Theorem I. The three Angles of every plain Triangle are equal to two Right ones, *viz.* A + B + C = 180.

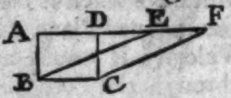
Theor. II. A Right Line EF croſſing two parallel Lines AB, and DC, makes the Angles G=H=I=K; and a=b=c=d.



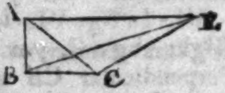
Theor. III. The outward Angle ACD =BAC+ABC, the two inward and oppoſite Angles.



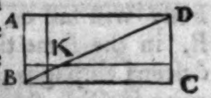
Theor. IV. Parallelograms ABCD, EBCF, on the ſame Baſe BC, and between the ſame Parallels BC and AF, are equal.



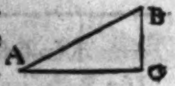
Theor. V. Triangles ABC, EBC, on the ſame Baſe BC, and between the ſame Parallels AE and BC, are equal.



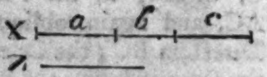
Theor. VI. In every Parallelogram ABCD, the Complements about the Diagonal BD, *viz.* AK and KC, are equal.



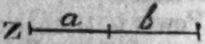
Theor. VII. In any Right-angled Triangle ABC, the Square of the Hypothenuſe AB, is equal to the Sum of the Squares of the other two Sides AC and BC, *viz.* $ABq = ACq + BCq$.



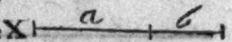
Theor. VIII. If there be two Lines X and Z, and X be divided into the Parts *a*, *b*, *c*, &c. we have, $X \times Z = Za + Zb + Zc$.



Theor. IX. If a Right Line Z be divided into two equal Parts, *viz.* $a = b$; then we have, $ZZ = Za + Zb$.



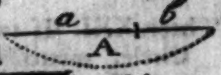
Theor. X. If the Right Line X be any how cut into *a*, *b*, we ſhall have $Xa = aa + ab$.



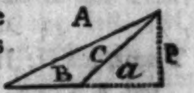
Theor. XI. The Line X divided as before, we ſhall have $XX = aa + 2ab + bb$.

Theor. XII. If any Right Line A, be divided into the equal Parts *a*, *a*, and another Line (*c*) be added thereto; we ſhall have $aa + 2ac + cc =$ Square of $a + c$.

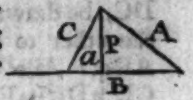
Theor. XIII. If a Right Line A be ſo divided into, *a*, *b*, that it be Ab=aa, it is then divided into extream and mean Proportion; and $a=\sqrt{AA+\frac{1}{4}AA}:-\frac{1}{2}A$.



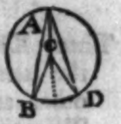
Theor. XIV. In the Obtuſe-angled Triangle ABC, reduced to a Right one, it is AA=BB+CC+2Ba.



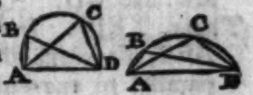
Theor. XV. In the Acute-angled Triangle ABC, the Perpendicular P being let fall; we have AA=BB+CC−2Ba.



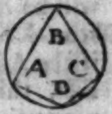
Theor. XVI. An Angle at the Center of a Circle, as BCD, is double to the Angle at the Periphery BAD ſtanding on the ſame Arch BD.



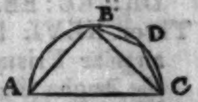
Theor. XVII. Any Angles ABD, ACD, in the ſame Segment of a Circle ABCD, are equal to each other.



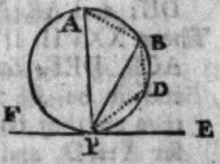
Theor. XVIII. The two oppoſite Angles, *viz.* A+C, or B+D, of any Quadrilateral Figure ABCD, inſcribed in a Circle, are equal to two Right Angles.



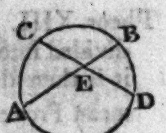
Theor. XIX. The Angle ABC in the Semicircle ABDC, is a Right one; but the Angle BAC in a greater Segment is leſs than a Right one, and the Angle BDC in a leſſer Segment is greater than a Right one.



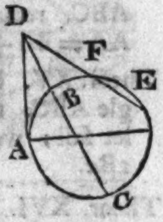
Theor. XX. If any Line FE touch the Circle in the Point P, and the Line BP be drawn from the Point of Contact, to cut the Circle, then the Angles BPE, BPF, which it makes with the Tangent Line, ſhall be equal to thoſe made in the Alternate Segments of the Circle, *viz.* the Angle BPE=BAP, and BPF=BDP.



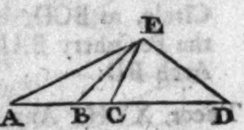
Theor. XXI. If two Lines AB, CD in a Circle, mutually interſect each other, as in E, then $AE \times EB = CE \times ED$.



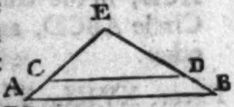
Theor. XXII. If from any Point D without a Circle, two Right Lines DA, DC, be drawn, one to touch the Circle, the other to cut it; then $CD \times DB = ADq$. Alſo draw another DE; then $CD \times DB = ED \times DF = ADq$.



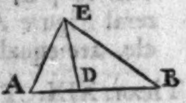
Theor. XXIII. Triangles that have the ſame Altitude, are to each other as their Baſes; thus the Triangle AEB : BEC : CED :: AB : BC : CD.



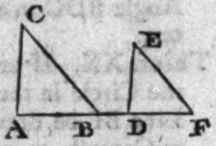
Theor. XXIV. In any Triangle AEB, if a Right Line CD be drawn parallel to one Side AB; then it will be AC : CE :: BD : DE.



Theor. XXV. In any Triangle AEB, if the Angle E be biſected by the Line ED, this Line ſhall cut the Baſe AB proportional to the Sides; for AD : DB :: AE : EB.

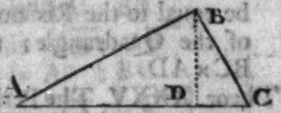


Theor. XXVI. If in the two Equiangular Triangles ACB, DEF, the Angles C = E, A = D, and B = F, then the Sides AC : CB :: DE : EF; and AB : AC :: DF : DE; and AB : BC :: DF : FE.

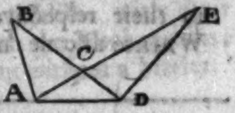


Theor. XXVII. If in two Triangles ACB, DEF, the Angle B = F, and it be AB : BC :: DF : FE; then are they both Equiangular; for A = D, and C = E.

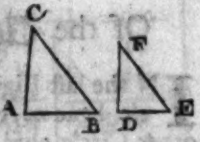
Theor.



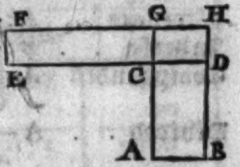
Theor. XXVIII. If from the Right Angle B, of the Triangle ABC, you let fall a Perpendicular BD to the Baſe; then will the Triangle ADB be ſimilar to the Triangle CDB, and both theſe ſimilar to the firſt ABC. For the Angle $A = CBD$, and $C = ABD$, and $AB : AD :: BC : CD :: AC : AB$; and $AD : DB :: DB : DC :: AB : BC$.



Theor. XXIX. Equal Triangles ABC, CDE, having one Angle ACB in one equal to one Angle DCE of the other, have the Sides about the equal Angles reciprocal, *viz.* $BC : CD :: EC : CA$. The ſame is true of Parallelograms.

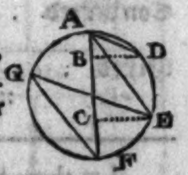


Theor. XXX. Similar Triangles ABC, DEF, having the Angle $B = E$, and the Sides $AB : AC :: DE : DF$; then will the Triangles be $ABC : DEF :: ABq : DEq$. The ſame is true of ſimilar Polygons.

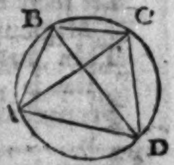


Theor. XXXI Equiangular Parallelograms AD, CF, have the Proportion to each other that is compounded of their Sides, *viz.*

$$\frac{AD}{CF} = \frac{AC}{CG} \times \frac{EC}{CD}.$$

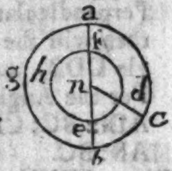


Theor. XXXII. In the Circle AEFG, the Angles $DAE : EAF :: DE : EF$. Therefore the Angle $EAF = EGF$, or any other ſtanding on the ſame Arch.



Theor. XXXIII. The Squares of any Number of Chords AD, AE, are as their verſed Sines, *i. e.* $ADq : AEq :: AB : AC$.

Theor. XXXIV. In any Quadrilateral Figure inſcribed in a Circle, as ABCD; the Rectangle under the two Diagonals AC, DB, will be equal to the Rectangles under the two oppoſite Sides of the Quadrangle; that is, AC × BD = AB × CD + BC × AD.



Theor. XXXV. The Periphery of Circles are in Proportion to their Diameters; thus acbg : ab :: fdeh : fe.

Theor. XXXVI. In concentric Circles, any two Semidiameters nb, nc, cut off the Arches bc, ed, which are ſimilar to their reſpective Circles, *i. e.* bgac : cb :: ehfd : de. Whence alſo ne : nb :: ed : bc.

SECT. XLVI.

Of the **Variation** of **Proportionals.**

IN the laſt Figure, ſuppoſe the greater Part of the Circle bgac = A, bc = B; alſo the greater Part of the other Circle ehfd = C, and ed = D. Then it will be

Directly . .	A : B :: C : D	16 : 4 :: 12 : 3
Alternate .	A : C :: B : D	16 : 12 :: 4 : 3
Inverſe . .	B : A :: D : C	4 : 16 :: 3 : 12
Compounded	A + B : B :: C + D : D	20 : 4 :: 15 : 3
Divided . .	A — B : B :: C — D : D	12 : 4 :: 9 : 3
Converted .	A : B ± A :: C : D ± C	$16 : \begin{cases} 20 \\ 12 \end{cases} :: 12 : \begin{cases} 15 \\ 9 \end{cases}$
Mixed . .	A + B : A — B :: C + D : C — D	20 : 12 :: 15 : 9

Let there be two Rows of Magnitudes . . .	A, B, C, D, E, F.
Then **Ordinate** Proportion is	A : B :: D : E. B : C :: E : F.
Perturbate Proportion is	A : B :: E : F. B : C :: D : E.
Proportion of **Equality** is	A : C :: D : F.

Duplicate Ratio of $A : B$ is $AA : BB$.
Subduplicate — $A : B$ is $\sqrt{A} : \sqrt{B}$
Triplicate —— $A : B$ is $A^3 : B^3$
Subtriplicate — $A : B$ is $\sqrt[3]{A} : \sqrt[3]{B}$.
Seſquiplicate — $A : B$ is $A\sqrt{A} : B\sqrt{B}$.

Note, The Theorems and Proportions contain'd in the two laſt Sections are all Demonſtrated in *Euclid*'s Elements; which I ſhall very ſhortly publiſh, with their Application to all the uſeful Parts of the Mathematics; in a Method new, and eaſy to be learnt without a Maſter.

TABLE I. Of Money.

Shilling.		Farthing . .	.02083	48
		Penny . . .	.083	12
One Pound Sterling.	4	Farthing . .	.0010416	960
	12	Penny . . .	.00416	240
	20	Shilling . .	.05	20

TABLE II. Troy Weight.

Ounce.		Grain . . .	.002083	480
		Pennyweight	.05	20
Pound.	24	Grain . . .	.000173	5760
	20	Pennyweight	.00416	240
	12	Ounce . . .	.083	12

TABLE III. Apothecaries Weight.

Ounce.		Grain . . .	.002083	480
		Scruple . .	.0416	24
		Dram . . .	.125	8
Pound.	20	Grain . . .	.000173	5760
	3	Scruple . .	.003472	288
	8	Dram . . .	.010416	96
	12	Ounce . . .	.083	12

TABLE IV. Averdupoiſe Weight.

Pound.		Dram . . .	.003906	256
		Ounce . . .	.0625	16
Hundred.		Dram . . .	.000035	28672
		Ounce . . .	.000558	1792
		Pound . . .	.008928	112
		Quarter . .	.25	4
Tun.	16	Dram . . .	.0000017	573440
	16	Ounce . . .	.000028	35840
	28	Pound . . .	.000446	2240
	4	Quarter . .	.0125.	80
	20	Hundred . .	.05.	20

TABLE V. Wine Meaſure.

Tun.	42	Gallon . . .	.003967	252
	1½	Tierce . . .	$.1\dot{6}$	6
	1⅓	Hogſhead . .	.25	4
	1½	Punchion . .	$.\dot{3}$	3
	2	Butt or Pipe	.5	2

TABLE VI. Ale, &c. Meaſure.

Hogſhead.	9	Gallon Beer	$0\dot{1}8\dot{5}$	54
	8	Gallon Ale	$.0208\dot{3}$	48
	8½	Gallon Com.	.019608	51
	2	Firkin . . .	$.1\dot{6}$	6
	2	Kilderkin .	$.\dot{3}$	3
	1½	Barrel . . .	$.\dot{6}$	1½

TABLE VII. Dry Meaſure.

Quarter.		Gallon . . .	.015625.	64
		Peck	.03125.	32
		Buſhel . . .	.125.	8
Load.	2	Gallon . . .	.003125.	320
	4	Peck	00625.	160
	8	Buſhel . . .	.025.	40
	5	Quarter . .	.2.	5

TABLE VIII. Long Meaſure.

Yard.		Barley Corn	.00905	180
		Inch	.027	36
		Foot	.3	3
Rod.		Inch	.005	198
		Foot	.06	16½
		Yard	.18	5½
Mile.	12	Inch	.000016	63360
	3	Foot	.000189	5280
	5½	Yard	.0005681	1760
	40	Rod	.003125.	320
	8	Furlong . .	.125.	8

TABLE IX. Of Time.

Day.		Second . . .	.0000011	86400
		Minute . . .	.000694	1440
		Hour . . .	.0416	24
Year.	60	Second . . .	.000000	31449600
	60	Minute . . .	.000002	524160
	24	Hour . . .	.000144	8736
	7	Day	.002739	364
	4	Week . . .	.019178	52
	13	Month . . .	.076712	13

TABLE X. Of Motion.

Sign.		Second . .	.0000902	108000
		Minute . .	.0005.	1800
		Degree . . .	.03.	30
Circle.	60	Minute . .	.000046	21000
	30	Degree . . .	.0027	360
	12	Sign . . .	.083.	12

Though the ten Tables above are common, yet their Form and Conſtruction, as they here ſtand, is new, and more advantageous than any other I could think of. In the firſt Column are contain'd the integral Quantities; in the ſecond, the Numbers, ſhewing how many of one Part of the Integer in the third Column make one of the next following; the fourth Column expreſſeth the Decimal Part of the Integer proper to each Denomination it ſtands againſt; and in the fifth Column, you have the Numbers, ſhewing how many of every Denomination in the third Column are contain'd in the Integer in the firſt Column.

TABLE XI. Of Square Meaſure.

Inches Square.						
144=	1 = Foot Square.					
1296=	9=	1 Yard Square.				
39204=	$272\frac{1}{4}$=	$30\frac{1}{4}$=	1 Pole Square.			
1568160=	10890=	1210=	40=	1 Rood Square.		
6272640=	43560=	4840=	160=	4=	1 Acre Square.	
4014489600=	27878400=	3097600=	102400=	2560=	640=	1 Mile Square.

TABLE XII. Of Solid Meaſure.

Solid Inches.						
1728=	1 Foot Solid.					
46656=	27 =	1 Yard Solid.				
7762329=	$4492\frac{1}{8}$=	$166\frac{3}{10}$=	1 Pole Solid.			
1963885176=	$1136507\frac{6}{18}$=	$42092\frac{8}{10}$=	253=	1 Rood Solid.		
15711081408=	9092061=	336743=	2024=	8=	1 Acre Solid.	
254358061056000=	14719795200=	5451776000=	32770584=	129528=	16191=	1 Mile Solid.

TABLE XIII. **Foreign Coins.**

		English Coin.		
		l.	s.	d.
French Coins.	A Denier.	0	0	0 3/40
	A Soulz.	0	0	0 9/10
	A Livre.	0	1	6
	A Crown.	0	4	6
Low Country Coins.	A Stiver.	0	0	1 1/5
	A *Flemish* Shilling.	0	0	7 1/5
	A Gilder.	0	2	0
	An Emblem Dollar.	0	2	3 3/5
	A Campen Dollar.	0	2	7 1/5
	A *Zeland* Dollar.	0	3	0
	A *Lyons* Dollar.	0	4	0
	A Specie Dollar.	0	5	0
	A Duccatoon.	0	6	3 3/5
German Coins.	A Rix Dollar of the Empire.	0	4	5 1/4
	A Gilder of *Noremberg*.	0	7	1
Italian and Spanish Coins.	A Livre at *Leghorn*.	0	0	9
	Florence Crown Currant.	0	5	3
	Venice Ducat de Banco.	0	4	4
	The Currant Ducat.	0	3	4
	The *Naples* Ducat.	0	5	0
	The *Cadiz* Ducat.	0	5	6 1/4
	The *Barcelona* Ducat.	0	6	0
	The *Valence*.	0	5	3
	The *Bergonia* Ducat.	0	4	4
	The *Portugal* Teſſoon.	0	1	3
	The Piece of Eight.	0	4	6

TABLE XIV. Foreign Weight.

London.	1.000 ℔ Averd.	Holland.	1.0526
Antwerp.	1.04	Lisbon.	1.135
Amſterdam.	1.1	Lyon.	0.9345
Abbeville.	1.0989	Leghorn.	0.75
Ancona.	0.78	Milan.	0.7
Avignon.	0.8928	Mirandola.	0.75
Bourdeaux.	1.0989	Norimberg.	1.1363
Burgundy.	1.0989	Naples.	0.71
Bolonia.	0.8	Paris.	1.1235
Bruges.	1.0204	Prague.	1.2048
Calabria.	0.73	Placentia.	0.72
Calais.	0.9345	Rochel.	0.8928
Diep.	1.0989	Rome.	0.7874
Dantzick.	0.862	Rouen.	1.1089
Ferrara.	0.75	Sevil.	0.9259
Florence.	0.78	Thoulouſe.	0.8928
Flanders.	0.9433	Turin.	0.82
Geneva.	1.07	Venice { Subtile.	0.64
Genoa { Subtile.	0.71	Groſs.	1.06
Groſs.	0.7	Vienna.	1.23
Hamburgh.	1.0865		

TABLE XV. Foreign Meaſure.

London. Foot =	1.000	Amſterdam. Ell =	2.269
Amſterdam.	0.942	Antwerp.	2.273
Antwerp.	0.946	Bavaria.	0.954
Brill.	1.103	Bologn.	2.056
Bononia.	1.204	Bononia.	2.147
Bremen.	0.964	Dantzick.	1.903
Cologn.	0.954	Florence.	1.913
Copenhagen.	0.965	Frankfort.	1.826
Dantzick.	0.944	Hamburgh.	1.905
Dort.	1.184	Leipſick.	2.26
Frankfort ad Mæn.	0.948	Lubeck.	1.908
Lorrain.	0.958	Lyons.	3.976
Mechlin.	0.919	Noremberg.	2.227
Mantua.	1.569	Rynland.	2.26
Middleburg.	0.991	Vienna.	1.053
Paris.	1.068	Spaniſh Palm. =	0.751
Prague.	1.026	Naples.	0.861
Riga.	1.831	Genoa.	0.83
Roman.	0.967	Spaniſh Vare. =	3.001
Greek.	1.007	Lisbon.	2.75
Spaniſh.	1.001	Gibraltar.	2.76
Strasburgh.	0.92	Toledo.	2.685
Toledo.	0.899	Parma Cubit. =	1.866
Turin.	1.062	China.	1.016
Venice.	1.162	Cairo.	1.824
		Turkiſh Pike. =	2.2
		Perſian Araſh. =	3.157

Table XVI. Of Scripture Meaſure of Length.

Eng.	Feet.	Inches.	Decimals.
1 Digit	0	0	.912
4 = 1 Palm.	0	3	.648
12 = 3 = 1 Span.	0	10	.944
24 = 6 = 2 = 1 Cubit.	1	9	.888
96 = 24 = 8 = 4 = 1 Fathom.	7	3	.552
144 = 36 = 12 = 6 = 1½ = *Ezekiel's* Reed.	10	11	.828
192 = 48 = 16 = 8 = 2 = 1⅓ = *Arabian* Pole.	14	7	.104
1920 = 480 = 160 = 80 = 20 = 13½ = 10 = Meaſ. Line.	145	11	.004

Table XVII. Of the Longer Scripture Meaſure.

Eng.	Miles.	Paces.	Feet. Dec
1 Cubit.	0	0	1.824
400 = 1 Stadium.	0	145	4.6
2000 = 5 = 1 Sabb. Day's Journey.	0	729	3
4000 = 10 = 2 = 1 Eaſtern Mile.	1	403	1
12000 = 30 = 6 = 3 = 1 Paraſang.	4	153	3
96000 = 240 = 48 = 24 = 8 = A Day's Journey.	33	172	4

Table XVIII. Of Jewiſh Weights.

	lb	oz.	pwt.	Gr.
1 Shekel.	00	00	09	2.57
100 = 1 Manch.	03	09	10	17 14
5000 = 50 = 1 Talent.	180	08	15	17.14

TABLE XIX. Of Jewiſh Money, reduced to *Engliſh* Coins.

	l.	*s.*	*d.*
1 Gerah.	00	00	1.36
10 = 1 Bekah.	00	01	1.7
20 = 2 = 1 Shekel.	00	02	3.37
1200 = 120 = 60 = 1 Minch or Mina.	06	16	10.5
60000 = 6000 = 3000 = 50 = 1 Talent of Silver.	342	03	9
A Solidus aureus, or Golden Shilling.	00	12	0.$\frac{1}{2}$
A Golden Shekel.	01	16	6
A Talent of Gold.	5475	00	0

TABLE XX. Of Jewiſh Dry Meaſure.

Corn Meaſure.	Pecks.	Galls.	Pints.	Sol. Inch.
1 Gachal.	0	0	0$\frac{7}{60}$	0.0013
20 = 1 Cab.	0	0	2$\frac{5}{6}$	0.12
36 = 1$\frac{4}{5}$ = 1 Omer.	0	0	5$\frac{1}{10}$	1.211
120 = 6 = 3$\frac{1}{3}$ = 1 Seah.	1	0	1	4.03
360 = 18 = 10 = 3 = 1 Epah.	3	0	3	12.11
1800 = 90 = 50 = 15 = 5 = 1 Lethech.	16	0	0	26.52
3600 = 180 = 100 = 30 = 10 = 2 = 1 Homer Choron.	32	0	1	19.04

TABLE XXI. Of Jewiſh Liquid Meaſure, reduced to Wine Meaſure.

	Galls.	Pints.	Sol. Inch.
1 Caph.	0	0$\frac{5}{8}$	0.15
1$\frac{1}{3}$ = 1 = Log.	0	0$\frac{3}{4}$	2.5
5$\frac{1}{3}$ = 4 = 1 Cab.	0	3	10.
16 = 12 = 3 = 1 Hin.	1	2	2.5
32 = 24 = 6 = 2 = 1 Seah.	2	4	5.
96 = 72 = 18 = 6 = 3 = 1 Bath or Epah.	7	4	15.
960 = 720 = 180 = 60 = 30 = 10 = 1 Coron or Homer.	75	5	7.

TABLE XXII. Of Meridional Parts to every 10 Min. of a Degree.

Deg. Lat.	0′	10′	20′	30′	40′	50′
0	0	10	20	30	40	50
1	60	70	80	90	100	110
2	120	130	140	150	160	170
3	180	190	200	210	220	230
4	240	250	260	270	280	290
5	300	310	320	330	340	350
6	360	370	380	390	400	410
7	421	431	441	451	461	471
8	481	491	501	511	521	532
9	542	552	562	572	582	592
10	603	613	623	633	643	653
11	664	674	684	794	704	715
12	725	735	745	755	766	776
13	786	797	807	817	827	838
14	848	858	869	879	889	900
15	910	920	931	941	951	962
16	972	983	993	1004	1014	1024
17	1035	1045	1056	1066	1077	1087
18	1098	1108	1119	1129	1140	1150
19	1161	1172	1182	1193	1203	1214
20	1225	1235	1246	1257	1267	1278
21	1289	1299	1310	1321	1332	1342
22	1353	1364	1375	1386	1396	1407
23	1418	1429	1440	1451	1462	1473
24	1484	1499	1505	1516	1527	1538
25	1549	1561	1572	1583	1594	1605
26	1616	1627	1638	1649	1661	1672
27	1683	1694	1705	1717	1728	1738
28	1751	1761	1773	1785	1796	1808
29	1819	1830	1842	1853	1865	1877
30	1888	1899	1911	1923	1934	1946
31	1958	1969	1981	1993	2004	2016
32	2028	2040	2052	2063	2075	2087
33	2099	2111	2123	2135	2147	2159
34	2171	2183	2195	2207	2219	2231
35	2244	2256	2268	2281	2293	[illegible]

Deg. Lat.	The foregoing Table continued.					
	0′	10′	20′	30′	40′	50′
36	2318	2330	2345	2355	2367	2380
37	2392	2405	2417	2430	2442	2455
38	2468	2481	2493	2506	2519	2532
39	2544	2557	2570	2583	2596	2609
40	2622	2635	2648	2662	2675	2688
41	2701	2714	2728	2741	2754	2768
42	2781	2795	2808	2822	2835	2849
43	2863	2876	2890	2904	2918	2931
44	2945	2959	2973	2987	3001	3015
45	3030	3044	3058	3072	3086	3101
46	3115	3130	3144	3159	3173	3188
47	3202	3217	3232	3247	3261	3276
48	3291	3306	3321	3336	3351	3366
49	3382	3397	3412	3428	3443	3459
50	3474	3490	3505	3521	3537	3553
51	3568	3584	3600	3616	3632	3649
52	3665	3681	3697	3714	3730	3747
53	3763	3780	3797	3814	3830	3847
54	3864	3881	3899	3916	3933	3950
55	3968	3985	4003	4020	4038	4056
56	4074	4092	4110	4128	4146	4164
57	4182	4201	4219	4238	4257	4275
58	4294	4313	4331	4351	4370	4390
59	4409	4428	4448	4468	4487	4507
60	4527	4547	4567	4588	4608	4629
61	4643	4670	4691	4711	4733	4754
62	4775	4796	4818	4839	4861	4883
63	4905	4927	4949	4972	4994	5017
64	5039	5062	5085	5018	5132	5155
65	5179	5203	5226	5250	5275	5299
66	5324	5348	5373	5390	5423	5449
67	5474	5500	5526	5552	5678	5704
68	5631	5658	5685	5712	5739	5767
69	5795	5823	5851	5879	5908	5937
70	5966	5996	6025	6055	6085	6115

Deg. Lat.	The foregoing Table continued.					
	0′	10′	20′	30′	40′	50′
71	6146	6177	6208	6239	6271	6303
72	6335	6368	6401	6434	6468	6501
73	6535	6570	6605	6640	6675	6718
74	6748	6783	6820	6857	6895	6933
75	6972	7010	7050	7089	7130	7170
76	7211	7253	7295	7338	7381	7424
77	7469	7513	7559	7605	7651	7698
78	7746	7795	7844	7894	7944	7996
79	8048	8100	8154	8209	8264	8320
80	8377	8435	8495	8555	8616	8678
81	8742	8806	8872	8939	9007	9077
82	9148	9221	9295	9371	9449	9528
83	9609	9692	9778	9865	9954	10046
84	10141	10238	10338	10441	10547	10656
85	10770	10887	11007	11133	11263	11398
86	11539	11686	11839	11999	12168	12344
87	12521	12718	12927	13150	13388	13644
88	13920	14221	14550	14914	15321	15783
89	16318	16950	17726	18729	20152	22623

TABLE

Table XXIII. Of Difference of Latitude and Departure to every Quarter-Point of the Compaſs, when the Diſtance exceeds not 10000.

Diff.	¼ Point.		½ Point.		¾ Point.		Whole Point		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	0.9988	0.0491	0.9952	0.0980	0.9892	0.1467	0.9808	0.1951	1 Point. 7.
2	1.9976	0.0981	1.9904	0.1960	1.9784	0.2935	1.9616	0.3902	
3	2.9964	0.1472	2.9856	0.2940	2.9675	0.4402	2.9424	0.5853	
4	3.9952	0.1963	3.9807	0.3921	3.9567	0.5869	3.9231	0.7804	
5	4.9940	0.2453	4.9759	0.4901	4.9459	0.7337	4.9035	0.9750	
6	5.9928	0.2944	5.9711	0.5881	5.9351	0.8804	5.8847	1.1705	
7	6.9916	0.3430	6.9663	0.6861	6.9242	1.0271	6.8655	1.3656	
8	7.9904	0.3925	7.9615	0.7841	7.9134	1.1738	7.8463	1.5607	
9	8.9892	0.4416	8.9567	0.8822	8.9026	1.3206	8.8271	1.7558	
1	0.9700	0.2430	0.9569	0.2903	0.9415	0.3369	0.9239	0.3827	2 Points. 6.
2	1.9401	0.4860	1.9139	0.5806	1.8831	0.6738	1.8478	0.7654	
3	2.9101	0.7289	2.8708	0.8709	2.8246	1.0107	2.7976	1.1480	
4	3.8801	0.9719	3.8278	1.1611	3.7662	1.3476	3.6964	1.5307	
5	4.8502	1.2149	4.7847	1.4514	4.7077	1.6844	4.6194	1.9134	
6	5.8202	1.4579	5.7416	1.7117	5.6493	2.0213	5.5433	2.2961	
7	6.7902	1.7009	6.6986	2.0320	6.5908	2.3582	6.4672	2.6788	
8	7.7602	1.9430	7.6555	2.3223	7.5324	2.6951	7.3910	3.0615	
9	8.7303	2.1868	8.6125	2.6126	8.4739	3.0320	8.3149	3.4441	
1	0.9040	0.4276	0.8819	0.4714	0.8577	0.5141	0.8315	0.5556	3 Points. 5.
2	1.8080	0.8551	1.7638	0.9428	1.7155	1.0282	1.6629	1.1111	
3	2.7120	1.2827	2.6458	1.4142	2.5732	1.5423	2.4944	1.6667	
4	3.6160	1.7102	3.5277	1.8856	3.4309	2.0564	3.3259	2.2223	
5	4.5199	2.1378	4.4096	2.3570	4.2886	2.5705	4.1573	2.7778	
6	5.4239	2.5053	5.2915	2.8284	5.1464	3.0846	4.9888	3.3334	
7	6.3279	2.9929	6.1734	3.2998	6.0041	3.5987	5.8203	3.8890	
8	7.2319	3.4204	7.0554	3.7712	6.8618	4.1128	6.6518	4.4446	
9	8.1359	3.8489	7.9373	4.2426	7.7196	4.6269	7.4832	5.0001	
1	0.8032	0.5957	0.7730	0.6344	0.7410	0.6716	0.7071	0.7071	4 Points.
2	1.6064	1.1914	1.5460	1.2688	1.4819	1.3431	1.4142	1.4142	
3	2.4096	1.7871	2.3190	1.9032	2.2229	2.0147	2.1213	2.1213	
4	3.2128	2.3828	3.0920	2.5376	2.9638	2.6862	2.8284	2.8284	
5	4.0160	2.9785	3.8650	3.1720	3.7048	3.3578	3.5355	3.5355	
6	4.8192	3.5742	4.6381	3.8064	4.4457	4.0294	4.2426	4.2426	
7	5.6224	4.1699	5.4111	4.4408	5.1867	4.7009	4.9497	4.9497	
8	6.4257	4.7656	6.1841	5.0751	5.9276	5.3725	5.6569	5.6569	
9	7.2289	5.3613	6.9571	5.7095	6.6686	6.0440	6.3640	6.3640	

The Uſe of this Table; Suppoſe the Courſe be $2\frac{1}{2}$ Points, and the Diſtance failed 7531 Miles: Quære the Difference of Latitude and Departure?

Set down the Parts of the Diſtance thus; and to each collect from the Table the Latitudes and Departures, which added, gives the Anwer.

Diſt.	Lat.	Dep.
7000	6173.4	3299.8
500	440.96	235.70
30	26.458	14.142
1	0.8819	0.4714
7531	6641.7	3550.1134

TABLE XXIV. Of the **Declinations**, and **Right Aſcenſions** reduced into Time, of 41 fixed Stars, for finding the Hour of the Night; and other Purpoſes of Aſtronomy, Navigation, *&c.*

Stars Names.	Declin.	Right Aſ. inMotion.	in Time.
	° ' "	° ' "	h ' "
chedir, in the Breaſt of *Caſſiopæa*. . .	52. 2. N20	6.18.30	0.25.14
olar Star.	87.54. 0	9.10.26	0.36.42
The Bright Star of *Aries*.	29. 9. 0	27.56.40	1.51.47
Mandibula, in the Jaw of the Whale. .	2.59. 32	42. 0. 0	2.48. 0
Algol, in the Head of *Meduſa*. . . .	39.53. 0	42.36.45	2.50.27
The Bright Star in the Side of *Perſeus*. .	48.51. 40	46.14.30	3. 4.58
The brighteſt of the *Pleiades*. . . .	23.14. 00	52.50. 0	3.31.20
ldebaran.	15.55. 46	65. 3. 0	4.20.12
apella, or Goat Star.	45.41. 50	74. 7.30	4.56.30
egulus, the Lyon's Heart.	13.17. 0	148.26. 7	9.53.44
orth Horn of the Bull.	28.20. 51	77.14. 0	5. 8.56
outh Horn.	20.56. 45	80.19.20	5.21.18
he Left Shoulder of *Orion*. . . .	6. 4. 28	77.37.25	5.10.22
he Right Shoulder of *Orion*.	7.19. 26	85. 4.38	5.40.18
aſtor.	32.27. 0	109.15.50	7.17. 3
ollux.	28.39. 14	112. 7.39	7.28.30
rocyon, or little Dog-Star.	5.54. 15	111.14.12	7.24.57
ubbe.	56. 0. 0	175.30. 0	11.42. 0
he Southermoſt of the two preceding Stars, in the Wain of the greater Bear. . .	57.50. 36	161.16.26	10.45. 6
TheNorthermoſt of them.	63.13. 40	161.37.50	10.46.31
he Tail of the Lyon, *Deneb*. . . .	16. 5. 30	173.46.10	11.35. 5

The

The foregoing Table continued.

Stars Names.	Declin.	Right Aſ. inMotion.	in Time
	° ′ ″	° ′ ″	h ′ ″
The firſt in the Tail of the great Bear. .	57.30. 0	190.23.00	12.41.3
The middlemoſt. . .	56.22. 36	197.44. 0	13.10.5
The laſt of the Three.	50.42. 0	204.11.50	13.36.4
Vindemiatrix. . . .	12.25. 40	192.30.52	12.46.
Arcturus.	20.38. 25	210.48.23	14. 3.1
The brighteſt in the Dragon's Head.	51.32. 0	267.35.45	17.50.2
Aſengue, in the Harp.	38.33. 10	276.54.20	18.27.3
The brighteſt Star in the Eagle. . .	8.10. 15	294.20. 0	19.37.2
Scheat, in the Flying-Horſe.	26.35. 2	342.37.20	22.50.2
The Head of *Andromeda*.	27.34. 22	358.33.30	23.54.1
The bright Star in the Tail of the Whale.	19.29. S 30	7.26.50	0.29.4
Rigel, in *Orion*'s left Foot.	8.33. 12	75.21. 0	5. 1.2
The middle Star in his Belt.	1.25. 0	80.34.31	5.22.1
The laſt in his Belt. .	2. 7. 12	81.44.40	5.26.5
Syrius, the great Dog-Star.	16.20. 58	98.16.40	6.33.
The *Hydra*'s Heart. .	7.29. 0	138.31.20	9.14.
The *Virgin*'s Spike. .	9.44. 30	197.42.28	13.10.5
The laſt but one in the *Hydra*'s Tail. .	21.43. 0	196. 2.27	13. 4.1
Antares, the *Scorpion*'s Heart.	25.47. 30	243.10.20	16.42.1
Fomahaunt, in the Mouth of the Southern Fiſh. . . .	31. 3. 30	340.35. 0	22.42.2

TABL

TABLE XXV. Of the Latitude and Longitude of the moſt eminent Places or Cities in the World.

Names of Cities.	Lat.		Long.	
	°	′	°	′
Aberdeen, *in* Scotland.	57	20	17	15
Aix la Chapelle, *in* Germany.	50	48	25	36
Alba Regalis, *in* Hungary.	47	23	39	24
Alicant, *in* Spain.	38	25	17	20
Achem, *in* Sumatra-Iſle.	5	30	112	30
Agra, *in* India.	26	30	98	20
Amaſia, *in* Natolia.	41	30	55	10
Aleppo, *in* Syria.	36	10	56	30
Algiers, *in* Barbary.	37	00	22	30
Amſterdam, *in* Holland.	52	29	24	00
Antwerp, *in the* Netherlands.	51	16	23	36
Archangel, *in* Ruſſia.	64	50	65	10
Arles, *in* France.	43	5	33	38
Aſſumption, *in* La Plata.	25	30	320 S	16
Aſtracan, *in* Tartary.	47	00	71	30
Ausburg, *in* Germany.	48	14	30	57
Bagdat, *in* Meſopotamia.	33	30	62	30
Bamberg, *in* Germany.	49	53	30	52
Barcelona, *in* Spain.	40	34	20	33
Baſil, *in* Switzerland.	47	34	27	16
Baſſora, *in* Arabia.	30	00	65	40
Belgrade, *in* Turkey.	45	20	42	34
Bergen, *in* Norway.	61	0	24	15
Bern, *in* Switzerland.	46	45	27	01
Berlin, *in* Germany.	52	33	33	52
Bologne, *in* Italy.	44	08	32	4
Borneo, *in Borneo Iſle.*	4	30	130	0
Boſton, *in* New-England.	42	30	308	30
Bourdeaux, *in* France.	44	50	17	50
Brandenburg, *in* Germany.	52	34	33	2
Briſtol, *in* England.	51	30	16	00
Brunſwick, *in* Germany.	52	36	30	34
Bruſſels, *in the* Netherlands.	50	54	23	36
Buda, *in* Hungary.	47	38	40	15
Cadiz, *in* Spain.	36	32	10	42
Cagliari, *in* Sardinia.	38	10	30	24

The foregoing Table continued.

Names of Cities.	Lat.		Long.	
	°	'	°	'
Calais, *in* France.	51	2	21	4
Calmar, *in* Swedeland.	57	6	36	32
Cambridge, *in* England.	52	17	18	50
Cambaya, *in* India.	23	30	90	30
Cairo, *in* Egypt.	29	40	51	30
Charles-Town, *in* Carolina.	34	0	300	0
Cologn, *in* Germany.	50	55	26	32
Conſtantinople.	43	0	54	20
Copenhagen, *in* Denmark.	56	13	32	30
Cracow, *in* Poland.	49	56	40	47
Dantzick, *in* Poland.	54	13	40	42
Dort, *in* Holland.			23	52
Dreſden, *in* Germany.	51	6	33	50
Dublin, *in* Ireland.	53	20	12	00
Dunkirk, *in* Flanders.	51	7	21	30
Edinburg, *in* Scotland.	56	7	15	50
Exeter, *in* England.	50	40	15	2
Florence, *in* Italy.	43	20	32	10
Frankfort, *on the Maine.*	50	3	28	12
Fez, *in* Barbary.	33	30	14	0
Fort St. George, India.	13	30	99	0
Geneva, *in* Italy.	46	21	25	38
Genoa, *in* Italy.	43	53	29	0
Gibraltar, *in* Spain.	35	59	11	32
Glaſcow, *in* Scotland.	55	50	14	30
Greenwich Obſervatory.	51	28½		
Hague, *in* Holland.	52	8	23	22
Hamburgh, *in* Denmark.	53	57	29	20
Hanover, *in* Germany.	52	35	29	36
Heidelberg, *in* Germany.	49	17	28	27
Jamaica, Middle.	18	25		
Jeruſalem, *in* Syria.	31	45	55	0
Iſpahan, *in* Perſia.	31	45	69	30
Kaffa, *in* Crim-Tartary.	47	20	61	20
Lima, *in* Peru.	23	30	296	40
Leghorn, *in* Italy.	42	52	31	00
Leyden, *in* Holland.	52	13	23	34

The foregoing Table continued.

Names of Cities.	Lat.		Long.	
	°	′	°	′
Leypſick, *in* Germany.	51	18	32	33
Liſbon, *in* Portugal.	38	45	7	37
London, *in* England.	51	32	18	36
Lubeck, *in* Denmark.	54	15	30	40
Lyons, *in* France.	45	24	24	8
Madrid, *in* Spain.	40	25	13	40
Magdeburg, *in* Germany.	52	17	32	0
Mantua, *in* Italy.	44	52	31	10
Majorca, *in* Majorca-Iſland.	39	4	20	33
Marſeilles, *in* France.	42	47	24	35
Mecca, *in* Arabia.	21	30	61	31
Mentz, *in* Germany.	49	57	27	43
Medina, *in* Arabia.	24	50	59	10
Mexico, *in* New Spain.	28	30	269	30
Middleburg, *in the* Netherlands.	51	35	22	48
Milan, *in* Italy.	44	55	29	13
Moſcow, *in* Ruſſia.	55	25	63	0
Morocco, *in* Barbary.	31	40	10	0
Monomotapa, *in* Africa.	24	30	38	30
Munſter, *in* Germany.	52	0	27	12
Nantes, *in* France.	47	13	16	48
Nanking, *in* China.	32	0	127	20
Naples, *in* Italy.	40	56	36	15
Negropont, *in* Turkey.	38	20	48	25
Nice, *in* Italy.	43	18	26	52
Norwich, *in* England.	52	45	20	10
Neurenburg, *in* Germany.	49	24	31	11
St. Omers, *in the* Netherlands.	50	52	21	22
Orleans, *in* France.	47	44	20	42
Oſnaburg, *in* Germany.	52	25	27	33
Oſtend, *in* Flanders.	51	18	22	8
Oxford, *in* England.	51	46	17	28
Padua, *in* Italy.	45	17	32	30
Palermo, *in* Sicily.	37	26	34	50
Paris, *in* France.	48	45	21	30
Parma, *in* Italy.	44	24	30	40

The foregoing Table continued.

Names of Cities.	Lat.		Long.	
	°	′	°	′
Peking, *in* China.	39	40	133	36
Pegu, *in* India.	17	10	115	30
Placentia, *in* Spain.	39	48	11	50
Porto, *in* Portugal.	41	18	8	14
Prague, *in* Bohemia.	49	58	34	33
Preſburg, *in* Hungary.	48	25	38	14
Quebeck, *in* Canada.	47	0	305	0
Raguſa, *in* Dalmatia.	43	3	40	40
Ratiſbon, *in* Germany.	48	55	32	18
Ravenna, *in* Italy.	44	6	33	10
Revel, *in* Finland.	59	13	46	32
Rome, *in* Italy.	41	51	34	20
Rotterdam, *in* Holland.	52	0	23	34
Roven, *in* France.	49	26	20	2
Saltzburg, *in* Germany.	47	37	33	32
Saragoſa, *in* Spain.	41	35	17	0
Seville, *in* Spain.	37	30	11	14
Siam, *in* India.	4	30	119	30
Sleſwick, *in* Denmark.	55	57	29	24
Smyrna, *in* Natolia.	38	0	46	50
Soiſſons, *in* France.	49	19	22	32
Spires, *in* Germany.	49	11	28	12
Stockholm, *in* Sweden.	59	26	39	5
Straſburg, *in* Germany.	48	28	27	26
Syracuſe, *in* Sicily.	36	18	37	20
Suratt, *in* India.	21	30		
Toledo, *in* Spain.	39	24	13	30
Tombut, *in* Negroland.	14	50	8	30
Toulouſe, *in* France.	43	29	19	48
Tours, *in* France.	47	23	19	18
Trent, *in* Germany.	45	48	31	24
Triers, *in* Germany.	49	50	26	0
Tripoli, *in* Syria.	34	10	55	30
Troy, *in* Natolia.	39	30	46	20
Tunis, *in* Barbary.	37	0	28	30
Turin, *in* Italy.	44	32	27	26
Valentia, *in* Spain.	39	25	17	15

The foregoing Table continued.

Names of Cities.	Lat. °	Lat. ′	Long. °	Long. ′
Valladolid, *in* Spain.	41	48	12	50
Venice, *in* Italy.	45	20	34	4
Verona, *in* Italy.	45	14	31	20
Vienna, *in* Germany.	48	14	37	5
Viterbo, *in* Italy.	42	11	33	48
Ulm, *in* Germany.	48	14	30	00
Uraniberg, Tycho's *Obſerv.*	55	54½		
Utrecht, *in the* Netherlands.	52	9	24	20
Warſaw, *in* Poland.	52	7	42	5
Waterford, *in* Ireland.	52	10	11	20
Wilna, *in* Poland.	54	31	47	14
Wirtemburg, *in* Germany.	51	54	33	2
Wolfenbuttle, *in* Germany.	52	28	39	30
Worms, *in* Germany.	49	32	28	0
York, *in* England.	54	0	17	46
Ypres, *in* Flanders.	50	57	22	3
Zagrab, *in* Sclavonia.	46	11	37	28
Zara, *in* Dalmatia.	44	34	37	20
Zurick, *in* Switzerland.	47	12	28	12

Theſe Latitudes and Longitudes are ſuch as I find them in Mr. *Echard*'s *Gazetteer*; but it were to be wiſhed, that a Table of more Truth and Exactneſs could be had.

The Uſe of this Table is to enable us readily to find any of thoſe Places in a Map, and to know the Difference of Meridians in Time for Purpoſes of Dialling, *&c.*

In order to which, you muſt underſtand, that 15 Degrees Longitude is equal to one Hour in Time; and ſo proportionally for Minutes.

But the following Table will ſhew this by Inſpection.

Note, The Longitude here is reckon'd from Mount *Teneriff*.

TABLE XXVI. For converting Time and Degrees, &c. mutually into each other.

Time. H. ' "	Mot. ° ' "	Time. ' " '''	Motion. ° ' "	Motion. ' " '''	Time. ' " '''	Motion. ° ' "	Motion. ' " '''	Time. ' " '''	Motion. ° ' "	Motion. ' " '''	Time. ' " '''	Motion. ° ' "	Motion. ' " '''
1	15	1	0	15	16	4	0	31	7	45	46	11	30
2	30	2	0	30	17	4	15	32	8	0	47	11	45
3	45	3	0	45	18	4	30	33	8	15	48	12	0
4	60	4	1	0	19	4	45	34	8	30	49	12	15
5	75	5	1	15	20	5	0	35	8	45	50	12	30
6	90	6	1	30	21	5	15	36	9	0	51	12	45
7	105	7	1	45	22	5	30	37	9	15	52	13	0
8	120	8	2	0	23	5	45	38	9	30	53	13	15
9	135	9	2	15	24	6	0	39	9	45	54	13	30
10	150	10	2	30	25	6	15	40	10	0	55	13	45
11	165	11	2	45	26	6	30	41	10	15	56	14	0
12	180	12	3	0	27	6	45	42	10	30	57	14	15
13	195	13	3	15	28	7	0	43	10	45	58	14	30
14	210	14	3	30	29	7	15	44	11	0	59	14	45
15	225	15	3	45	30	7	30	45	11	15	60	15	0
16	240												
17	255												
18	270												
19	285												
20	300												
21	315												
22	330												
23	345												
24	360												

The Uſe of the Table: Suppoſe I would know in what Time th Motion of 158° 17′ 39″ in the Equinoctial was perform'd;

	Mot.			H.	'	"	
In the Table you find againſt.	150°	0'	0"	10	0	0	
	8	15	0	0	33	0	
		2	30	0	0	10	
			9	0	0	0	36'''
Anſwer	158.	17.	39.	10.	33.	10.	36. Ti

TABLE XXVII. Shewing the Golden Number, and Epacts.

Gol. N.	Epact.	Anno Domini.				Gol. N.	Epact.	Anno Domini.		
7	17	1735	1754	1773	1792	17	7	1745	1764	1783
8	28	36	55	74	93	18	18	46	65	84
9	9	37	56	75	94	19	29	47	66	85
10	20	38	57	76	95	1	11	48	67	86
11	1	39	58	77	96	2	22	49	68	87
12	12	40	59	78	97	3	3	50	69	88
13	23	41	60	79	98	4	14	51	70	89
14	4	42	61	80	99	5	25	52	71	90
15	15	43	62	81		6	6	53	72	91
16	26	44	63	82						

Seek the Year of our Lord, and right againſt it on the Left Hand, you have the Golden Number and *Engliſh* Epact under their proper Titles for the ſaid Year. *Note,* if from the *Engliſh* Epact you ſubduct 11, there will remain the *Roman* Epact.

TABLE XXVIII. Shewing the Cycle of the Sun and Dominical Letter in both Accounts, to the Year 1800.

Cycle ☉.	Dom. Let. *Engliſh.*	Dom. Let. *Roman.*	Anno Domini.			Cycle ☉.	Dom. Let. *Engliſh.*	Dom. Let. *Roman.*	Anno Domini.		
1	GF	DC	1728	1756	1784	15	CB	G	1742	1770	1798
2	E	B	29	57	85	16	B	F	43	71	99
3	D	A	30	58	86	17	AG	ED	44	72	1800
4	C	G	31	59	87	18	F	C	45	73	
5	BA	FE	32	60	88	19	E	B	46	74	
6	G	D	33	61	89	20	D	A	47	75	
7	F	C	34	62	90	21	CB	GF	48	76	
8	E	B	35	63	91	22	A	E	49	77	
9	DC	AG	36	64	92	23	G	D	50	78	
10	B	F	37	65	93	24	F	C	51	79	
11	A	E	38	66	94	25	ED	BA	52	80	
12	G	D	39	67	95	26	C	G	53	81	
13	FE	CB	40	68	96	27	B	F	54	82	
14	D	A	41	69	97	28	A	E	55	83	

TABLE XXIX. Shewing the Number of Direction for ever.

Gol. Numb.	A	B	C	D	E	F	G
1	19	20	21	22	16	17	18
2	5	6	7	8	9	10	11
3	26	27	28	29	30	24	25
4	19	13	14	15	16	17	18
5	5	6	7	8	2	3	4
6	26	27	21	22	23	24	25
7	12	13	14	15	16	10	11
8	33	34	35	29	30	31	32
9	19	20	21	22	23	24	18
10	5	6	7	8	9	10	4
11	26	27	28	29	30	31	32
12	19	20	21	15	16	17	18
13	5	6	7	8	9	10	4
14	26	27	28	29	23	24	25
15	12	13	14	15	16	17	18
16	5	6	7	1	2	3	4
17	26	20	21	22	23	24	25
18	12	13	14	15	9	10	11
19	33	34	28	29	30	31	32

TABLE XXX. Shewing what Day of the Week any Day of the Month is for ever.

Julian M.	Days of the Week.							*Gregorian* M.
Jan. Oct.	A	B	C	D	E	F	G	Apr. July.
Feb.Mar.Nov.	D	E	F	G	A	B	C	Auguſt.
April July.	G	A	B	C	D	E	F	Sep. Dec.
May	B	C	D	E	F	G	A	October.
June	E	F	G	A	B	C	D	Feb.Mar.Nov.
Auguſt.	C	D	E	F	G	A	B	Jan. May.
Sep. Dec.	F	G	A	B	C	D	E	June.
Days of the Month.	1 8 15 22 29	2 9 16 23 30	3 10 17 24 31	4 11 18 25	5 12 19 26	6 13 20 27	7 14 21 28	Days of the Month.

The Uſe of Table XXX. What Day of the Week is *May* 19, 1735?

Seek the Dominical Letter for that Year, which is E, then againſt *May* find E, and caſting your Eye down that Column, you'll ſee (amongſt the Days of the Month) that the 4, 11, 18, 25, Days of *May*, are *Sundays*; therefore the 5, 12, 19, 26 are *Mondays*, and ſo on; and thus you proceed for any other.

TABLE XXXI. Of Moveable Feaſts.		
Septuageſima-Sunday.	*Jan.*	17
Shrove-Sunday.	*Feb.*	0
Firſt Day of Lent.	*Feb.*	3
Midlent-Sunday.	*Mar.*	0
Eaſter-Day.	*Mar.*	21
Eaſter-Term begins.	*Apr.*	7
Rogation-Sunday.	*Apr.*	25
Aſcenſion-Day.	*Apr.*	29
Eaſter-Term ends.	*May*	3
White-Sunday.	*May*	9
Trinity-Sunday.	*May*	16
Trinity-Term begins.	*May*	21
Ends.	*June*	9

The Uſe of this Table: On what Month and Day thereof will Eaſter fall in the Year 1735? Proceed thus, firſt find the Number of Direction; then to the Day in the

Table. . . . . *March* 21

Add the Number of Direction. 16

The Sum is 37

From which ſubduct the Number of Days in *March* 31

There will remain for Anſwer. . . . . . *April* 6

Thus do for any other Feaſt.

PROB. I. To find the Moon's Age.

Add the Epact, the Day of the Month, and Number of Months (from *March*, to the Month you reckon to incluſively) together; and the Sum, if under 30, is the Moon's Age; if above 30 or 60, deduct them.

Example. I would know the Moon's Age this very Day, *January* 1, 1734?

Add { 6 = Epact for 1734.
1 = The Day of the Month.
11 = The Months from *March* incluſive.

The Sum is 18 = The Moon's Age.

Note, From *January* the firſt, to *March* the firſt, you muſt uſe the Epact of the preceding Year.

PROB.

PROB. II. To find the New and Full Moons.

Add the Epact and Number of Months from *March* together, and ſubſtract the Sum from 30, or 59, the Remainder is the Day of the New Moon; to which add 15 for the Day of the Full Moon.

Let the next New and Full Moon be required from this Day; as in the Example above . . . } 6 = Epact.
11 = Number of Months.

The Sum 17
Which ſubduct from 30

There remains the Day of the New Moon. . . } 13th of *January*.
Add 15

The Sum is the Day of the Full Moon. . . . } 28, for the Mean Motion.

PROB. III. To find the Moon's Southing.

Multiply her Age by 4, divide that Product by 5, the Quotient is the Hour; the Remainder is twelve Times ſo many Minutes.

Example, for this firſt of *January*, 1734, as above: her Age $18 \times 4 = 72$; then $5)72(= 14$, or 2^h $24'$ in the Morning.

70
2
12
24 = Minutes.

Or thus, multiply her Age by 8, cut off the firſt Figure to the Right Hand, which multiply by 6 for Minutes, the Left Hand Figures being the Hour; thus, $18 \times 8 = 14{,}4$ and $4 \times 6 = 24$. Her Southing is 14^h $24'$ as before.

TABLE XXXII. Shewing the Sun's Place, Declination, Right Aſcenſion, Amplitude, Riſing and Setting, Beginning and End of Twilight, Time when due Eaſt and Weſt, and the Equation of Time to every fifth Day of the Month.

Months.	Days.	Sun's Place.		Declination.		Right Aſcenſion.		Amplitude from E and W to N and S.		Riſing and Setting	Beg. and End of Twilight.	Time when due Eaſt or Weſt.	Clocks, &c. too faſt, or ſlow.	
		°	'	°	'	°	'	°	'	H ' H	H ' H	H ' H	'	"
January.	1	21 ♑	56	21 S	42	293	30	37	6	3. 1.4	5.54. 7	4.46.8	8 F	50
	6	27	1	20	48	299	3	34	45	7.54.5	5.48. 7	4.49.8	10	38
	11	2 ♒	7	19	44	304	26	32	52	7.47.5	5.42. 7	4.54.8	12	7
	16	7	11	18	30	309	43	30	38	7.39.5	5.36. 7	4.58.8	13	20
	21	12	16	17	10	314	40	28	20	7.30.5	5.30. 7	5. 3.7	14	11
	26	17	20	15	40	319	46	25	45	7.22.5	5.21. 7	5. 8.7	14	40
February.	1	23	24	13	45	325	36	22	28	7.11.5	5.13. 7	5.15.7	14	50
	6	28	26	12	2	330	35	19	3	7. 2.5	5. 4. 7	5.21.7	14	38
	11	3 ♓	28	10	15	335	28	16	34	6.52.6	4.55. 8	5.27.7	14	6
	16	8	29	8	25	340	7	13	34	6.42.6	4.46. 8	5.33.7	13	20
	21	13	30	6	30	344	47	10	29	6.32.6	4.36. 8	5.39.7	12	20
	26	18	29	4	34	349	32	7	22	6.23.6	4.25. 8	5.45.7	11	6
March.	1	21	26	3	23	352	7	5	32	6.17.6	4.19. 8	5.49.7	10	8
	6	26	27	1	20	356	45	2	20	6. 7.6	4. 8. 8	5.55.7	8	34
	11	1 ♈	24	0 N	34	1	20	0	55	5.57.7	3.59. 9	6. 2.6	7	3
	16	6	21	2	31	6	18	4	6	5.48.7	3.45. 9	6. 8.6	5	28
	21	11	17	4	28	10	21	7	10	5.38.7	3.33. 9	6.14.6	3	54
	26	16	12	6	22	14	58	10	22	5.28.7	3.20. 9	6.21.6	2	23
April.	1	22	4	8	36	20	20	13	56	5.17.7	3. 4. 9	6.28.6	0	42
	6	26	57	10	24	24	57	16	59	5. 7.7	2.51.10	6.33.6	0 S	36
	11	1 ♉	50	12	7	29	40	19	46	4.58.8	2.35.10	6.40.6	1	48
	16	6	41	13	46	34	20	22	32	4.49.8	2.19.10	6.45.6	2	40
	21	11	32	15	19	39	4	25	10	4.40.8	2. 2.10	6.50.6	3	24
	26	16	22	16	45	43	52	27	36	4.32.8	1.47.11	6.56.6	3	54

The foregoing Table continued.

Months.	Days.	Sun's Place.		Declination.		Right Aſcenſion.		Amplitude from E and W to N and S.		Riſing and Setting.	Beg. and End of Twilight.	Time when due Eaſt or Weſt.	Clocks, &c. too faſt, or ſlow.	
		°	′	°	′	°	′	°	′	H ′ H	H ′ H	H ′ H	′	″
May.	1	21	11	18	5	48	40	29	57	4.24.8	1.23.11	7. 0.5	4	16
	6	26	0	19	17	53	45	32	6	4.16.8	0.58.12	7. 5.5	4	11
	11	0 ♊	48	20	21	58	42	34	0	4. 8.8	0.14.12	7. 9.5	3	58
	16	5	36	21	16	63	36	35	40	4. 2.8	No	7.12.5	3	34
	21	10	23	22	2	68	45	37	8	3.57.9		7.15.5	2	54
	26	15	10	22	39	73	50	38	15	3.53.9		7.18.5	2	6
June.	1	20	54	23	10	80	7	39	13	3.49.9	dark Nights.	7.20.5	0	59
	6	25	40	23	24	85	21	39	37	3.48.9		7.20.5	0 F	3
	11	0 ♋	26	23	29	90	28	39	40	3.47.9		7.21.5	1	7
	16	5	12	23	24	95	30	39	34	3.48.9		7.20.5	2	11
	21	9	58	23	7	100	45	39	6	3.50.9		7.20.5	3	10
	26	14	44	22	40	106	0	38	18	3.52.9		7.17.5	4	4
July.	1	19	30	22	4	111	4	37	8	3.57.9		7.15.5	4	47
	6	24	16	21	18	116	7	35	45	4. 3.8		7.12.5	5	20
	11	29	2	20	24	121	10	34	4	4. 9.8	0.19.12	7. 9.5	5	40
	16	3 ♌	40	19	20	126	0	32	10	4.16.8	0.59.12	7. 5.5	5	46
	51	8	36	18	9	131	2	30	3	4.23.8	1.24.11	7. 0.5	5	36
	26	13	23	16	50	135	50	27	47	4.30.8	1.44.11	6.56.6	5	12
Auguſt.	1	19	9	15	7	141	37	24	4	4.39.8	2. 8.10	6.48.6	4	25
	6	23	58	13	34	146	15	22	10	4.49.8	2.25.10	6.45.6	3	30
	11	28	47	11	46	151	10	19	20	4.58.8	2.40.10	6.39.6	2	25
	16	3 ♍	37	10	12	155	36	16	30	5. 7.7	2.55.10	6.33.6	1	6
	21	8	27	8	26	160	4	13	3	5.17.7	3. 9. 9	6.27.6	0 S	21
	26	13	18	6	35	164	34	10	34	5.27.7	3.23. 9	6.20.6	1	58
September.	1	19	9	4	19	170	0	6	50	5.38.7	3.36. 9	6.13.6	3	53
	6	24	2	2	23	174	34	3	50	5.48.7	3.49. 9	6. 8.6	5	40
	11	28	56	0	26	179	0	0	41	5.57.7	4. 0. 8	6. 1.6	7	23
	16	3 ♎	50	1 S	30	183	30	2	19	6. 7.6	4.10. 8	5.55.7	9	1
	21	8	46	3	28	188	6	5	36	6.17.6	4.21. 8	5.49.7	10	37
	26	13	42	5	2	192	40	8	42	6.26.6	4.31. 8	5.42.7	12	5

The

The foregoing Table continued.

Months.	Days.	Sun's Place.		Declination.		Right Aſcenſion.		Amplitude from E and W to N and S.		Riſing and Setting.	Beg. and End. of Twilight.	Timewhendue Eaſt or Weſt.	Clocks, &c. too faſt, or ſlow.	
		°	'	°	'	°	'	°	'	H ' H	H ' H	H ' H	'	"
October.	1	18	39	7	19	197	15	11	47	6.37.6	4.42. 8	5.36.7	13	24
	6	23	37	9	11	201	52	14	42	6.47.6	4.52. 8	5.30.7	14	28
	11	28	36	10	59	206	32	17	48	6.57.6	5. 0. 7	5.25.7	15	16
	16	3 ♏	36	12	44	211	20	20	44	7. 6.5	5. 9. 7	5.18.7	15	48
	21	8	37	14	23	216	15	23	30	7.14.5	5.17. 7	5.13.7	16	1
	26	13	38	15	57	221	13	26	17	7.24.5	5.24. 7	5. 7.7	15	55
November.	1	19	40	17	40	227	11	29	12	7.34.5	5.34. 7	5. 1.7	15	21
	6	24	43	18	58	232	20	31	36	7.42.5	5.40. 7	4 56.8	14	28
	11	29	47	20	8	237	31	33	40	7.49.5	5.46. 7	4.52.8	13	15
	16	4 ♐	51	21	8	242	57	35	30	7.56.5	5.52. 7	4.48.8	11	43
	21	9	55	21	58	248	7	37	3	8. 2.4	5.55. 7	4.45.8	9	54
	26	15	0	22	38	253	40	38	15	8. 6.4	5.58. 7	4.41.8	7	44
December.	1	20	5	23	6	259	7	39	10	8.10.4	6.10. 6	4.40.8	5	30
	6	25	11	23	23	264	45	39	40	8.12.4	6.12. 6	4.40.8	3	5
	11	0 ♑	16	23	29	270	20	39	49	8.13.4	6.13. 6	4.41.8	0	36
	16	5	22	23	22	275	47	39	38	8.12.4	6.12. 6	4.41.8	1 F	54
	21	10	28	23	4	281	20	39	4	8.10.4	6.10. 6	4.42.8	4	18
	26	15	34	22	34	286	52	38	6	8. 6.4	6. 6. 6	4.43.8	6	34

Note, This Table is adjuſted to the preſent Year 1735, and wil ſerve for many Years to come, without any ſenſible Error.

F I N I S.

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cimal Arithmetick in all the *Parts* or *Branches* of *Arithmetical Science*; viz. *Vulgar Arithmetick*, *Vulgar Fractions*, *Duodecimal* and *Sexageſimal Arithmetick*; alſo in *Algebra* and *Logarithms*. In all which its *Excellency* and abſolute *Neceſſity* is fully evinced. III. Its Application and Uſe in all ſuch *Parts* of the *Mathematicks* as abſolutely require its *Aſſiſtance*; viz. *Plain Trigonometry*, and the Arts depending thereon; as, *Navigation*, *Fortification*, *Altimetry*, *and Longimetry*: Alſo the *Menſuration* of all Kinds of *Superficies and Solid Bodies*; and the Arts reſulting therefrom; as, *Gauging*, *Surveying*, &c. IV. *A New and Compleat Sett of Decimal Tables*, never before publiſhed; ſhewing by Inſpection the Value of all Kinds of Decimals (without the tedious Methods of Reductions hitherto uſed) to four or ſix Places of Figures: Alſo all the *Common Tables*, very much *inlarged*, *corrected*, *and improved*; wherein all the *Circulating Numbers are marked*. With all other Tables of Intereſt, Annuities, Exchange, *&c. neceſſary* to render the Work *compleat*. V. An exact and accurate *Canon of Logarithms* for natural Numbers. And thro' the whole, ſeveral Things new and uſeful, not here expreſs'd. By *Benj. Martin.*

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1. By the Logarithmetick Canon.
2. By Natural Sines, Tangents, *&c.*
3. By the Lord *Neiper*'s five circular Parts.
4. By the Sector.
5. By the Sliding Rule.
6. By Scale and Compaſſes.
7. By the Globe or Sphere.
8. By the Stereographic Projection.
9. By the Orthographic } Planiſphere.
10. By the Stereographic } Planiſphere.

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